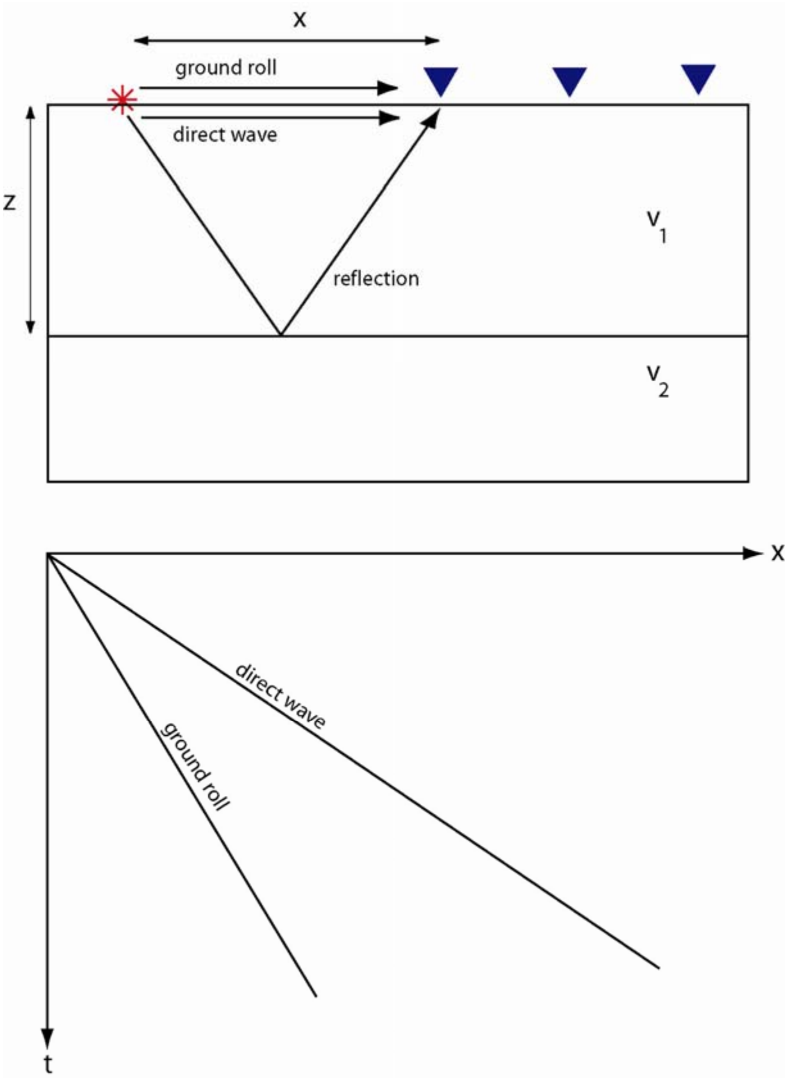
C2.1 Travel time curve for a single horizontal interface



Seismic energy can travel from the shot (*) to the receivers (geophones) by the 3 distinct routes shown in the upper panel. The lower panel is a **travel-time curve** and is a plot of travel time as a function of distance from the shot (x).

(1) Direct wave

A P-wave travels in a straight line just below the surface of the Earth. The travel time is $t_d = x / v_1$ which plots as a straight line on the travel time curve. This line passes through the origin on the travel time curve ($x = 0$ and $t = 0$).

(2) Ground roll

A Rayleigh wave travels in a straight line along the surface at a velocity = v_R. The travel time is $t_g = x / v_R$ which plots as a straight line on the travel time curve. This line passes through the origin ($x = 0$ and $t = 0$). Since $v_R < v_1$ note that **slower** seismic signals have a **steeper** slope on this plot.

(3) Reflection

A P-wave reflects from the interface between layer 1 and layer 2. The angle of incidence and reflection are equal (see C1.5). Using Pythagoras' Theorem, the distance travelled by the seismic signal on the downward leg of the journey is **:**

$$d = \sqrt{z^2 + \frac{x^2}{4}}$$

From symmetry, the total distance travelled is *2d*. The whole journey is travelled at velocity v_1, so the travel time is given by

$$t_{ref} = 2\frac{\sqrt{z^2 + \frac{x^2}{4}}}{v_1} = \frac{\sqrt{4z^2 + x^2}}{v_1}$$

Note that:

(1) When $x = 0$, t_{ref} is not zero, in contrast to the direct wave and ground roll.

(2) t_{ref} has a minimum value when $x = 0$. In this situation, the seismic signal travels **vertically** and makes an angle of 90° with the interface. This geometry is called **normal incidence** and the travel time is $t_{ref} = t_0 = 2z / v_1$

(3) In practice, it is impossible to measure t_0 since a geophone will be destroyed if place close to the shot. Observations at a range of x values can be used to extrapolate to find t_{ref} at $x = 0$. The travel time can be written:

$$t_{ref}^2 = t_0^2 + \frac{x^2}{v_1^2}$$

Thus a graph of t_{ref}^2 versus x^2 can be used for this. This will be a straight line with intercept t_0^2 when $x = 0$

(4) At normal incidence, two unknown model parameters (z and v_1) determine t_0. The solution of this **inverse problem** (to find the model parameters z and v_1) is non-unique. This is because we have two unknowns and just one equation.

(5) As x gets large the path taken by the reflection becomes very close to that taken by the direct wave. This means that the travel times for the reflection and direct wave become very close and $t_{ref} \rightarrow x / v_1$

Can you sketch the **travel-time curve** for the **reflection** on the figure above?

To address the problem of non-uniqueness in (3) and find the depth (z) and velocity (v_1) we need to consider values of $x > 0$.

$$t_{ref}^2 = \frac{4z^2}{v_1^2} + \frac{x^2}{v_1^2} = t_0^2 + \frac{x^2}{v_1^2}$$

Note that this equation is for a parabola (t_{ref} varies as x squared). Simple re-arrangement gives :

$$t_{ref} = \frac{\sqrt{4z^2 + x^2}}{v_1} = \frac{2z}{v_1}\sqrt{1 + (\frac{x}{2z})^2} = t_0\sqrt{1 + (\frac{x}{2z})^2}$$

$$t_{ref} = t_0\left[1 + (\frac{x}{2z})^2\right]^{1/2} = t_0\left[1 + (\frac{x}{v_1 t_0})^2\right]^{\frac{1}{2}}$$

We can simplify the equation for t_{ref} by using a power series expansion (Taylor's Theorem) and assuming that $x / v_1 t_0$ is relatively small.

$$t_{ref} = t_0\left[1 + (\frac{x}{v_1 t_0})^2\right]^{\frac{1}{2}} = t_0\left[1 + (\frac{x}{v_1 t_0})^2\right]^{\frac{1}{2}} = t_0\left[1 + \frac{1}{2}(\frac{x}{v_1 t_0})^2 + ...\right]$$

If the higher order terms are ignored, then we can write that:

$$t_{ref} = t_0\left[1 + \frac{1}{2}(\frac{x}{v_1 t_0})^2\right] = t_0 + \frac{x^2}{2v_1^2 t_0}$$

Re-arranging gives an expression for $t_{ref} - t_0$ which is termed the **normal moveout**.

$$t_{ref} - t_0 = \frac{x^2}{2v_1^2 t_0}$$

Normal moveout (NMO) is a measure of the **extra time** taken for seismic signal to travel on a non-vertical path, compared to the time for a signal travelling vertically. The word **normal** refers to the seismic energy travelling at **right angles** to the interface.

A graph of NMO versus x^2 will be a straight line, passing through the origin. The slope of the line will be $1/2v_1^2 t_0$. As x increases, the approximation made above becomes invalid and a deviation from the straight line will be observed.

Example 1

In this example, we will look at the synthetic seismic reflection data and compute the depth of the interface and estimate the velocity of the layer. These data are shown in graphical and numerical form on pages 4 and 5.

(1) Read the value of t_0 from the table t_0 = ________________

(2) Look at the geophone at x = 90 m. t_{ref} = ________________

(3) At this geophone, compute the NMO NMO = ________________

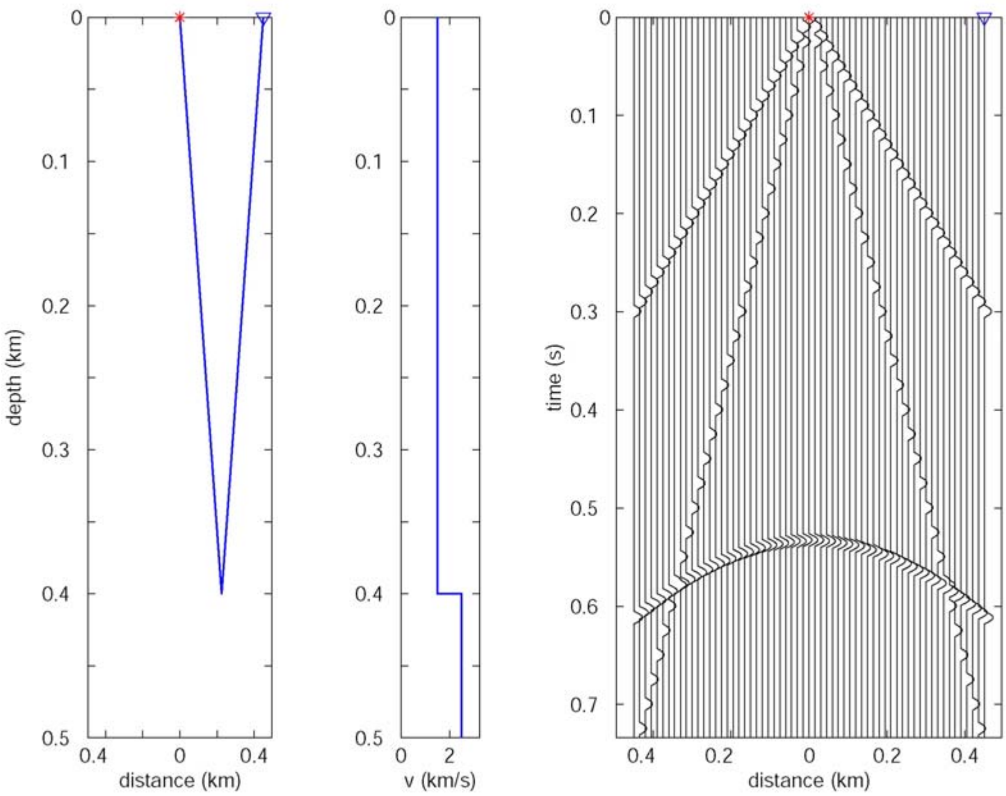
(4) Find v_1 using the equation above v_1 = ________________

(5) Using the values for v_1 and t_0 find the depth of the interface. This can be done with the equation for travel time at normal incidence $t_0 = 2z/v_1$. Rearrange this equation to find the depth to the interface.

z = __________________

Does your answer agree with that shown in the plot below? If not, can you suggest a reason for the discrepancy?

(6) What can we determine from the seismic data about the seismic velocity below the interface (v_2)?



Reflected amplitudes modelled assuming normal incidence
Attenuation, geometric spreading or AVO effects not included
Direct wave and ground roll included

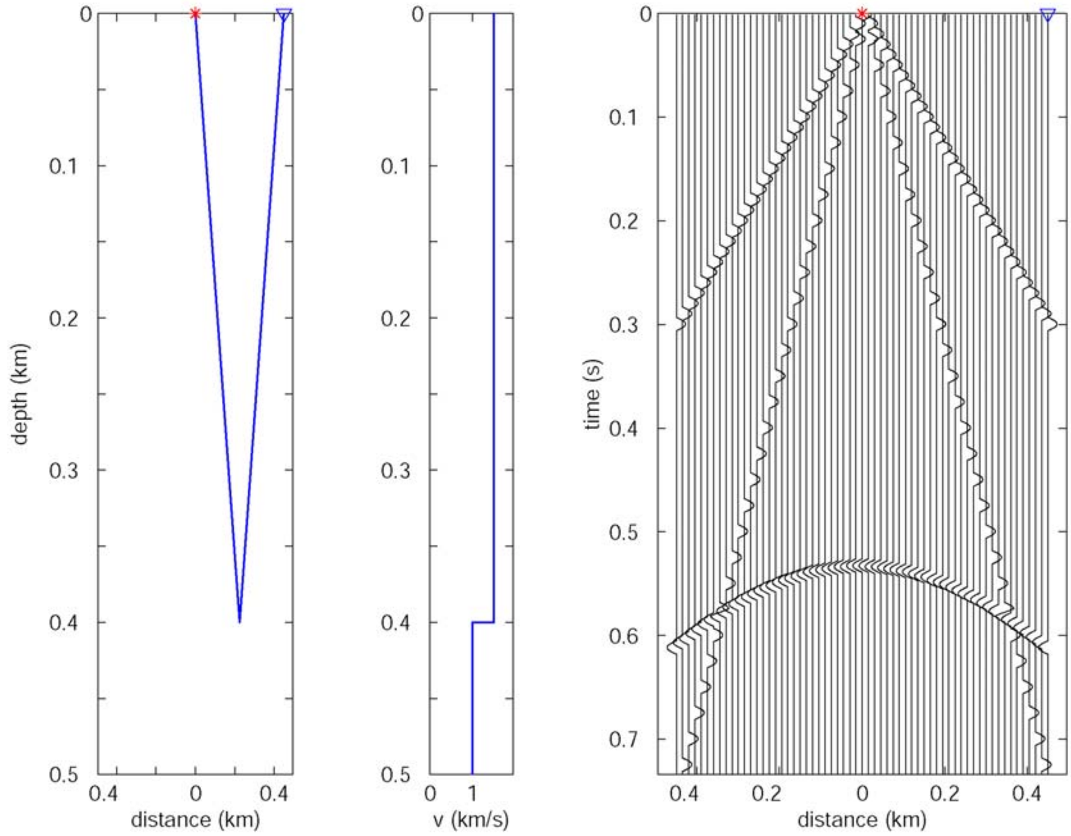
Numerical data in file *travel-time-1layer-ex1*

Note : To be more rigorous when computing v_1 it is best to plot a graph of *NMO* vs. x^2 which should be a straight line with slope $1/2v_1^2t_0$. A non-straight line may indicate that the simple form of the NMO equation is invalid.

C2.1 1-reflection, example 1

x(m)	ref1(ms)
-390.000	593.337
-375.000	589.021
-360.000	584.856
-345.000	580.822
-330.000	576.922
-315.000	573.200
-300.000	569.596
-285.000	566.172
-270.000	562.891
-255.000	559.773
-240.000	556.819
-225.000	554.029
-210.000	551.407
-195.000	548.951
-180.000	546.666
-165.000	544.561
-150.000	542.628
-135.000	540.876
-120.000	539.299
-105.000	537.911
-90.000	536.699
-75.000	535.671
-60.000	534.832
-45.000	534.177
-30.000	533.709
-15.000	533.427
0.000	533.333
15.000	533.427
30.000	533.707
45.000	534.177
60.000	534.832
75.000	535.671
90.000	536.699
105.000	537.905
120.000	539.299
135.000	540.868
150.000	542.628
165.000	544.561
180.000	546.666
195.000	548.951
210.000	551.394
225.000	554.029
240.000	556.819
255.000	559.773
270.000	562.891
285.000	566.172
300.000	569.596
315.000	573.180
360.000	584.833
375.000	589.021
390.000	593.337

C2.1 Shot gather for a single horizontal reflector
Example 2



Reflected amplitudes modelled assuming normal incidence
Attenuation, geometric spreading or AVO effects not included
Direct wave and ground roll included

Numerical data in file *travel-time-1layer-ex2*

(1) Look at the seismic reflection data presented above. How are they different from those in Example 1?

(2) Which model parameters (v_1, v_2 and z) will be different?

C2.1 Reflection from a single interface

MATLAB script : raytrace_v3