Fundamentals of Electric Circuits

CHAPTER 4

CIRCUIT THEOREMS

Dr. Ibrahim Hamdan

Circuit Theorems - Chapter 4

4.3 Superposition

4.4 Source Transformation

Note:- read these parts form Ref (*Fundamentals of Electric Circuits* 'Charles K. Alexander & Matthew N. O. Sadiku ')

Superposition Theorem

4.3 Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

4.3 Superposition Theorem

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using any techniques.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

4.3 Superposition Theorem

<u>Things to keep in mind:</u>

1. When turn off all other independent sources:
   - ➢ Independent voltage sources are replaced by 0 V (short circuit) and
   - ➢ Independent current sources are replaced by 0 A (open circuit).

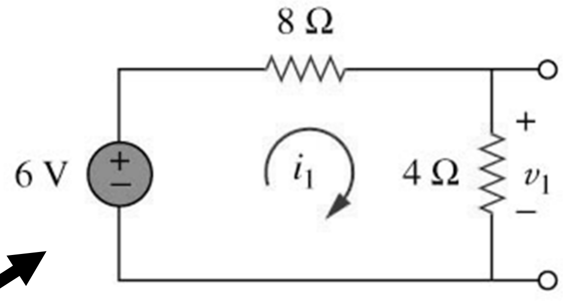
2. Dependent sources <u>are left</u> intact because they are controlled by circuit variables.

Examples for Superposition Theorem

4.3 Superposition Theorem

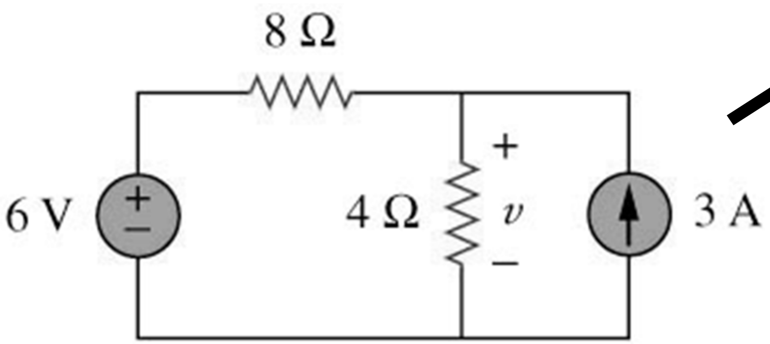
Example

Use the superposition theorem to find v in the circuit shown below.

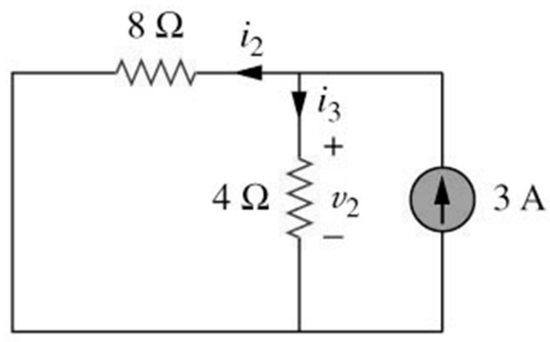


(a)

3A is discarded by open-circuit



6V is discarded by short-circuit



(b)

We consider the effects of 3A and 6V one by one, then add the two effects together for final v_o.

Use the superposition theorem to find v in the circuit of Fig. 4.6.

Example 4.3

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5\text{ A}$$

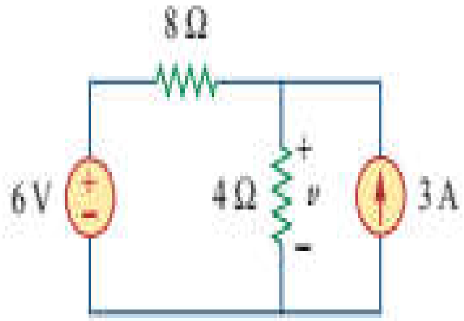


Figure 4.6
For Example 4.3.

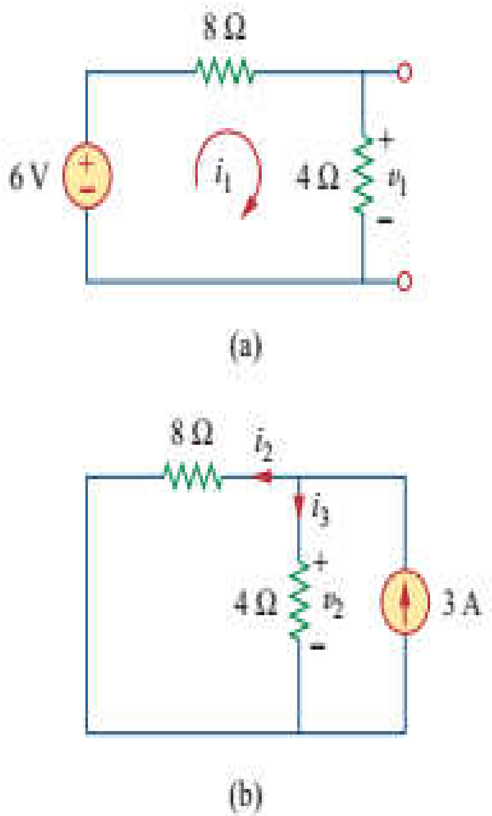


Figure 4.7
For Example 4.3: (a) calculating v_1, (b) calculating v_2.

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

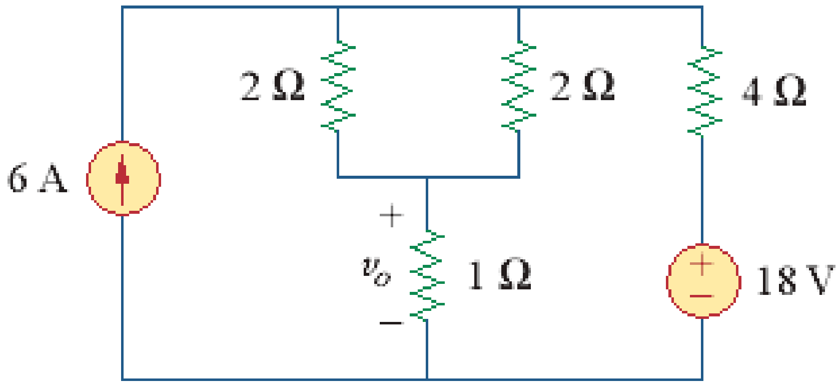
And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

4.3 Superposition Theorem

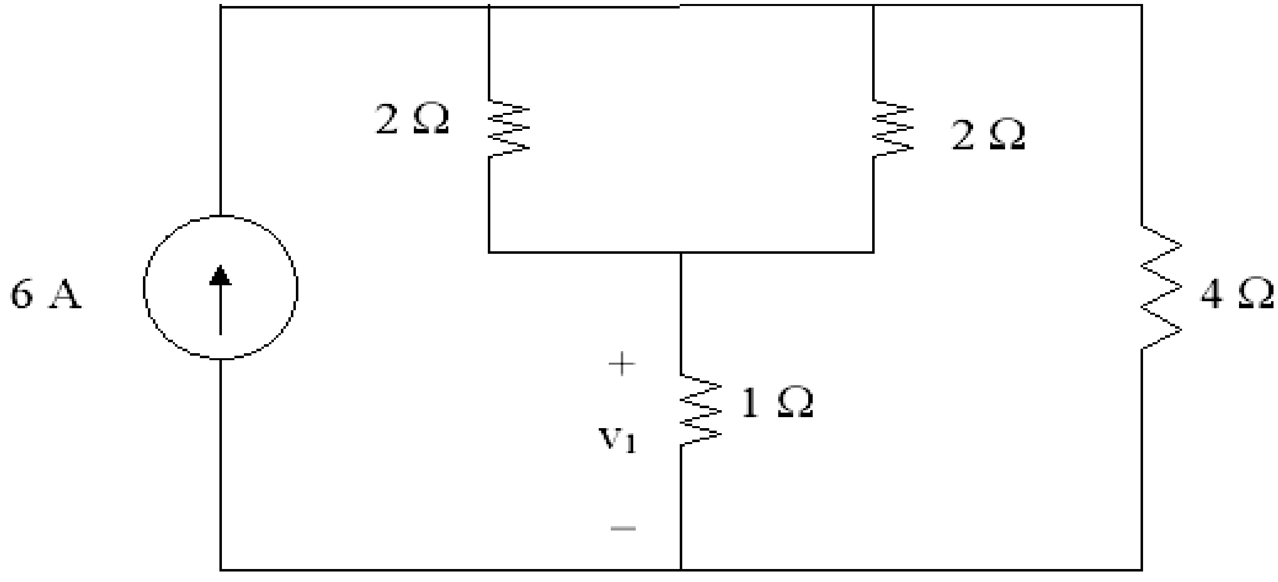
Example 1

Use superposition to find v_o in the circuit.



Solution

Let $v_o = v_1 + v_2$, where v_1 and v_2 are due to 6-A and 20-V sources respectively.
We find v_1 using the circuit below.

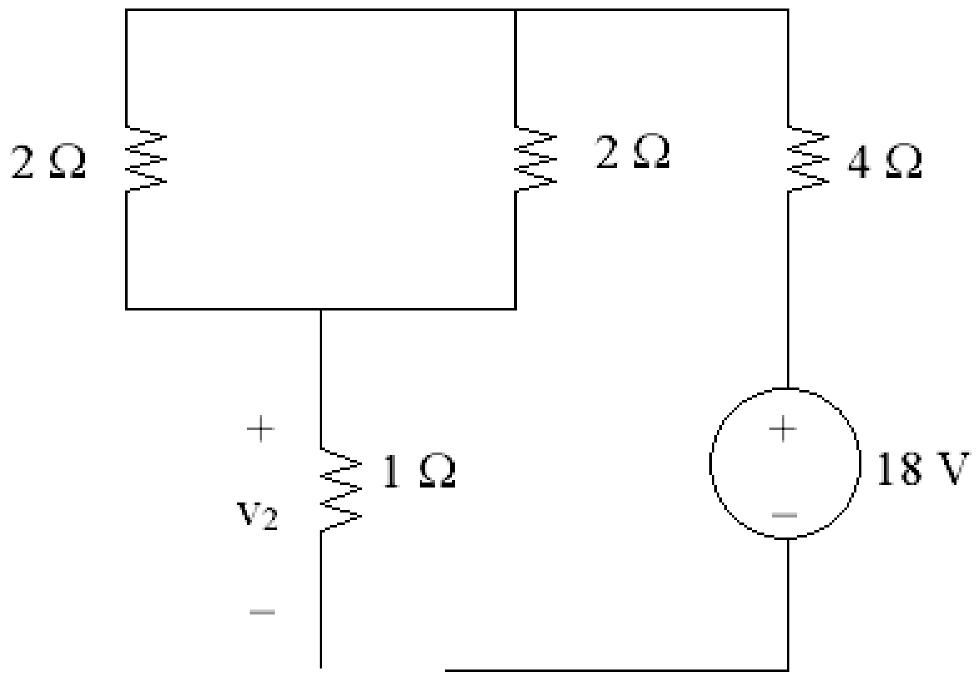


4.3 Superposition Theorem

$2//2 = 1\ \Omega,\qquad v_1 = 1\times\frac{4}{4+2}(6A) = 4\ V$

We find v_2 using the circuit below.

$$v_2 = \frac{1}{1+1+4}(18) = 3\ V$$



$v_o = v_1 + v_2 = 4 + 3 = \underline{\mathbf{7\ V}}$

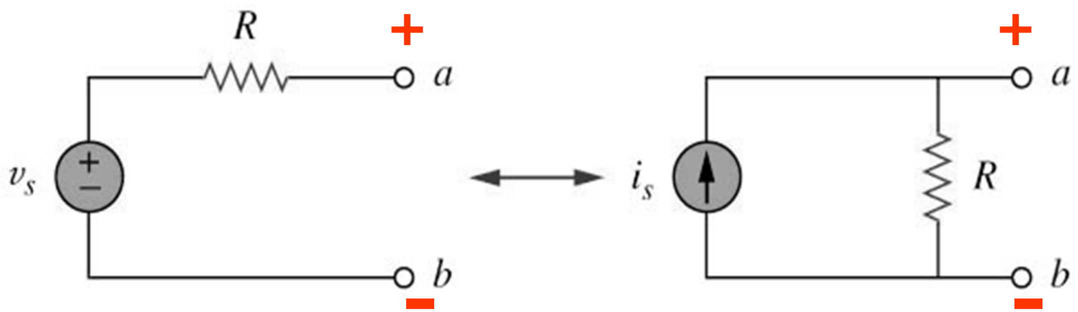
Source Transformation

4.4 Source Transformation

- An equivalent circuit is one whose *v-i* characteristics are identical with the original circuit.
- It is the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa.

4.4 Source Transformation

- The arrow of the current source is directed toward the positive terminal of the voltage source.

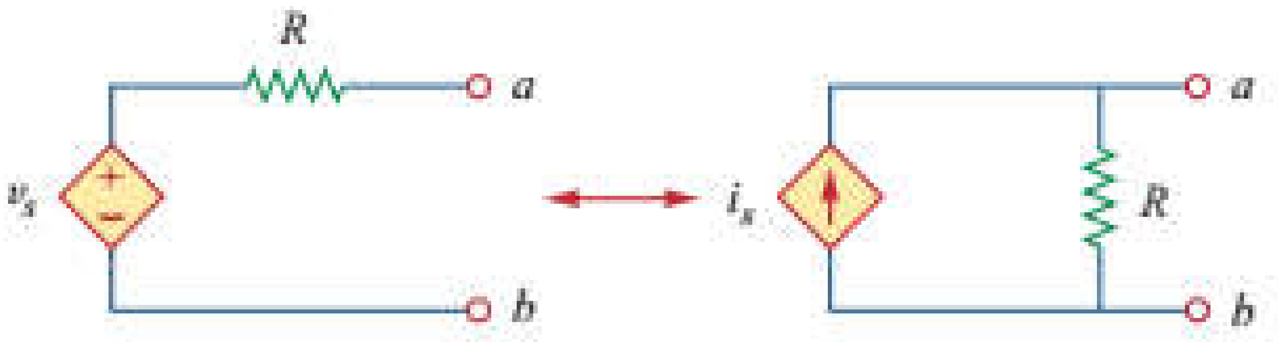


Independent source transform

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

4.4 Source Transformation

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.



Examples for source transformation

Use source transformation to find v_o in the circuit of Fig. 4.17.

Example 4.6

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

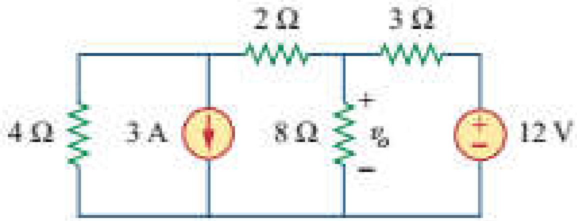


Figure 4.17
For Example 4.6.

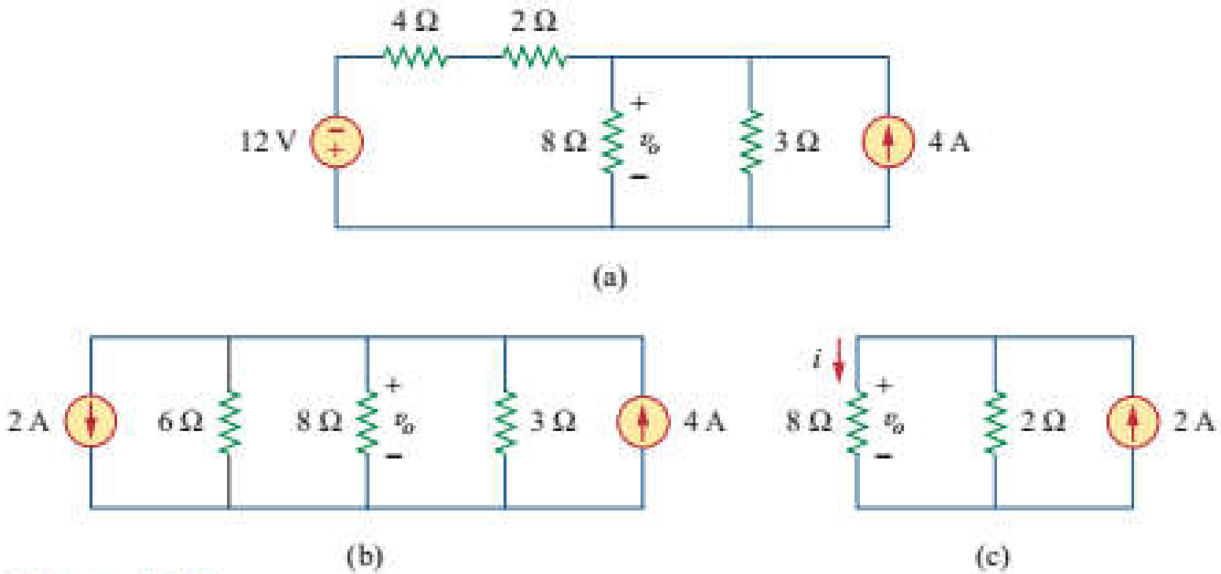


Figure 4.18
For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

and

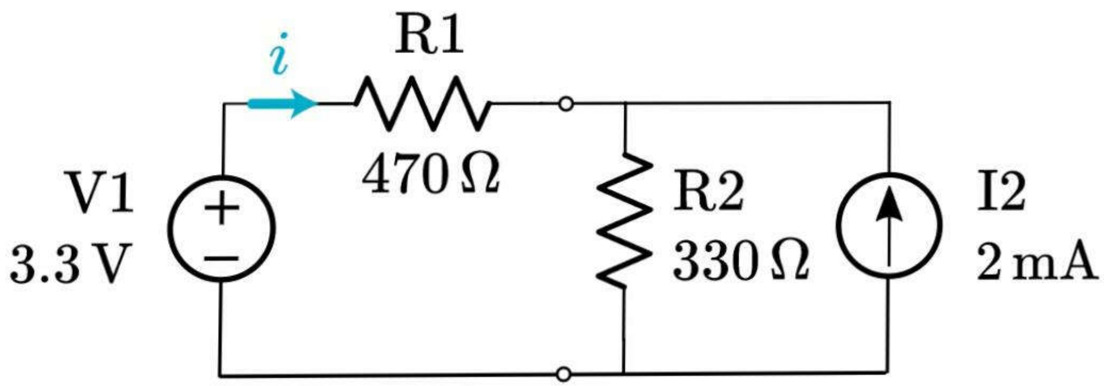
$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

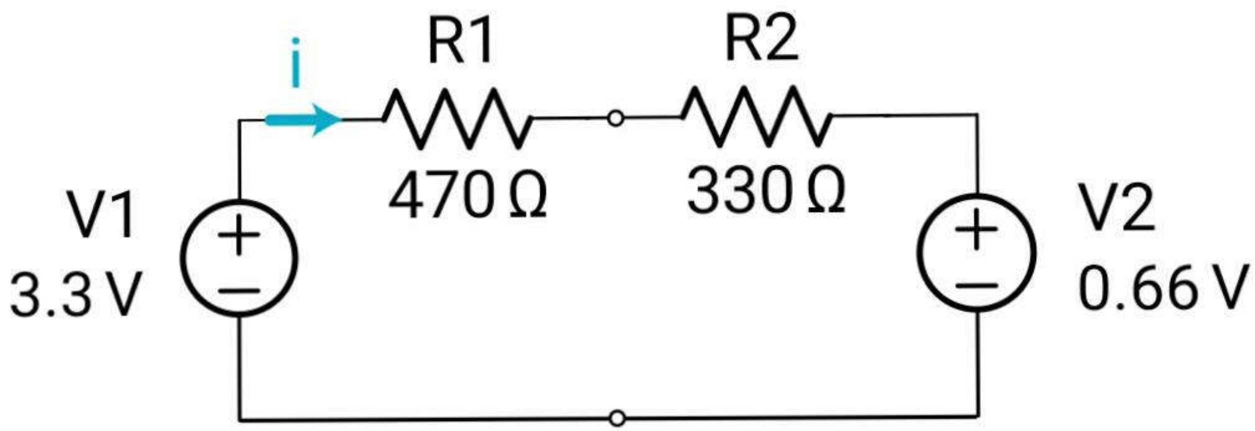
$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Example

- Find *i* (with source transformation)



- Transform the circuit form to the equivalent form
- **R2** is the same for both .
- **R2 = 330Ω**
- The voltage sources is
- **V2=i2 R2=2mA 330Ω=0.66V**



- Source transformation gave us two resistors in series. The voltage across the series resistors is **V1 V2 .**
- Ohm's Law gives us,
- **i=(V1 V2)÷(R1+R2)**

- ***i*=(3.3 0.66)÷(470+330)**

- <u>**i =3.3mA**</u>