Power System Voltage Stability (Lec. 4)

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Voltage Stability Indices

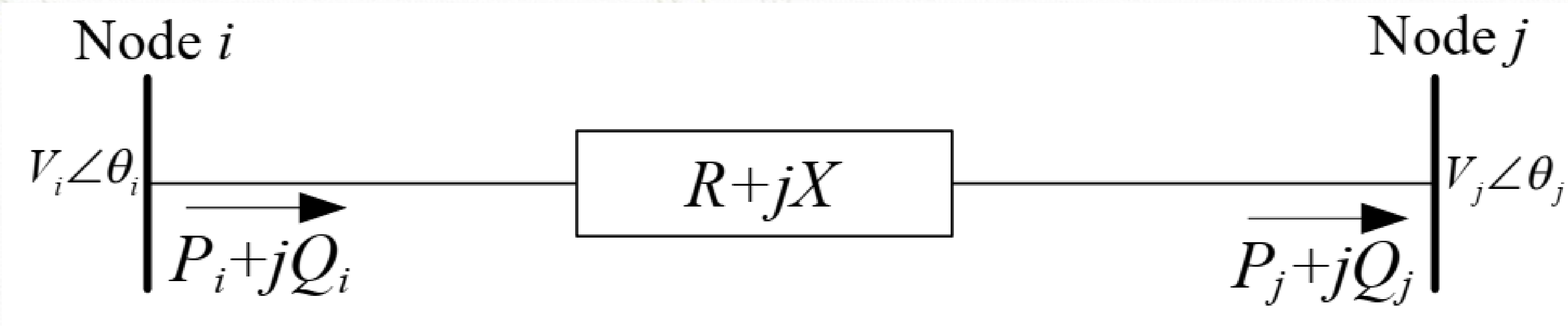
Introduction

- Voltage stability assessment is a major issue in monitoring the power system stability.
- Different voltage stability indices (VSIs) have been proposed for voltage stability assessment.
- These indices can be used for distributed generation (DG) placement and sizing, detecting the weak lines and buses and triggering the countermeasures against voltage instability.

Characteristics of the voltage collapse point

- All of VSIs have been derived from the characteristics of the voltage collapse point.
- Thus, investigating the characteristics of voltage collapse point can lead to a better understanding of the VSIs.
- Therefore, the characteristics of voltage collapse point are examined here.
- To explain what happens when the voltage collapse point is reached, the two bus representation of a power system is considered.

Characteristics of the voltage collapse point



- For simplicity, the shunt admittances are neglected.
- The current flows in the line segment calculated at nodes *i* and *j* is calculated as follows:

$$|I_i|^2 = \frac{P_i^2 + Q_i^2}{V_i^2} \qquad |I_i|^2 = \frac{P_j^2 + Q_j^2}{V_j^2}$$

Characteristics of the voltage collapse point

$$P_{loss} = \frac{P_j^2 + Q_j^2}{V_j^2} R$$

$$Q_{loss} = \frac{P_j^2 + Q_j^2}{V_j^2} X$$

- Then,

$$P_j = P_i - \frac{P_j^2 + Q_j^2}{V_j^2} R$$

$$Q_j = Q_i - \frac{P_j^2 + Q_j^2}{V_j^2} X$$

$$\frac{P_j^2 + Q_j^2}{V_j^2} = \frac{1}{V_i^2}\left[\left(P_j + \frac{P_j^2 + Q_j^2}{V_j^2} R\right)^2 + \left(Q_j + \frac{P_j^2 + Q_j^2}{V_j^2} X\right)^2\right]$$

Characteristics of the voltage collapse point

- Then,

$$V_i^2 = V_j^2 + 2(P_j R + Q_j X) + \left(\frac{P_j^2 + Q_j^2}{V_j^2}\right)(R^2 + X^2)$$

- Multiply each side by $V^2 j$ then:

$$V_i^2 V_j^2 = V_j^4 + 2(P_j R + Q_j X)V_j^2 + \left(P_j^2 + Q_j^2\right)(R^2 + X^2)$$

$$V_j^4 + [2(P_j R + Q_j X) - V_i^2]V_j^2 + \left(P_j^2 + Q_j^2\right)(R^2 + X^2) = 0$$

Characteristics of the voltage collapse point

- Rearranging the last equation gives an expression for the power-flow equation at node j:

$$aV_j^4 + bV_j^2 + c = 0$$

Where:

$$a = 1$$

$$b = 2(P_j R + Q_j X) - V_i^2$$

$$c = \left(P_j^2 + Q_j^2\right)(R^2 + X^2)$$

Characteristics of the voltage collapse point

- The solution of this equation can be directly determined as:

$$V_j^2 = \frac{\sqrt{b^2 - 4ac}}{2a}$$

- The visible solution of the terminal voltage at node j is obtained by applying this condition:

$$b^2 - 4ac \geq 0$$

Characteristics of the voltage collapse point

- At the voltage collapse point, last equation has two pairs of real identical roots (the Jacobian matrix is singular).
- If the load is increased further, then the roots become complex with real and imaginary parts.
- On the other hand, for the voltage to be stable, the discriminant of the equation must be greater than or equal to zero.
- In practice, the receiving end active and reactive powers are a function of receiving end voltage

Characteristics of the voltage collapse point

- The influence of load models on the roots of the last equation has been studied and it has been indicated that if both active and reactive powers are square functions of voltage $P_r = (V_r / V_n)^2 \; P_n$ and $Q_r = (V_r / V_n)^2 \; Q_n$, then there are no limits on the value of the receiving end voltage (i.e. the voltage collapse does not occur).
- At the limit, when the discriminant is equal to zero, it gives maximum load ability of line as

$$S_{r\,\max} = \frac{V_s^2}{4Z\cos^2\left((\theta - \varphi)/2\right)}$$

Characteristics of the voltage collapse point

- According to the maximum power transfer theorem, the following conditions are also satisfied at the maximum transferable power:
  - 1. Load absorbs maximum power.
  - 2. Thevenin and load impedance are equal in magnitude.
  - 3. The amplitude of the voltage drop across the Thevenin impedance is equal to the amplitude of load voltage.

Voltage stability indices

- The VSIs are classified based on the following ways:
  - 1. Jacobian matrix and system variables based VSIs
  - 2. Bus, line and overall VSIs
- Jacobian matrix based VSIs can calculate the voltage collapse point and determine the voltage stability margin. **But:**
- Computation time is high.
- Any topological change leads to change the Jacobian matrix and this matrix must be recalculated.

Voltage stability indices

- Hence, they are not suitable for real-time voltage stability assessment.
- The disadvantage of these indices is that they cannot accurately estimate the VSM so they can just present critical lines and buses.
- In many applications such as the first step of DG placement and sizing problems, VSIs are used to detect the weakest bus and line of power system.
- Therefore, the classification of VSIs according to the bus, line and over all indices can be very useful.

Line voltage stability indices

Line voltage stability indices

- Voltage stability analysis can be evaluated by the voltage stability index referred to a line.
- All of the line VSIs are formulated based on the two bus representation of a system as the previous figure where the shunt admittances are neglected.
- So, the theoretical base of most of the line VSIs are the same and the difference is in the assumptions used in each index.
- In proving most of the line VSIs, the discriminant of the voltage quadratic equation is set to be greater than or equal to zero to achieve the stability

Line voltage stability indices

1. Fast voltage stability index (FVSI)

- The FVSI based on the concept in which the discriminant of the voltage quadratic equation is set to be greater than or equal to zero.
- For a typical transmission line, the FVSI is calculated by:

$$FVSI = \frac{4Z^2 Q_r}{V_s^2 X}$$

Line voltage stability indices

1. Fast voltage stability index (FVSI)

- The FVSI must be below1 for a stable transmission line.
- If FVSI goes beyond 1.00, one of the buses that is connected to the line will experience a sudden voltage drop leading to system collapse.

2. Line Stability Index(L_{mn})

Line stability index, L_{mn} is obtained using the same concept as FVSI in which the discriminant of the voltage quadratic equation is set to be greater than or equal to zero.

Line voltage stability indices

2. Line Stability Index(L_{mn})

- For a typical transmission line, the L_{mn} calculated by

$$L_{mn} = \frac{4XQ_r}{(V_s \ \sin(\theta - \delta))^2}$$

- In the L_{mn}, the effect of the active power on the voltage stability as well as the line shunt admittance are neglected. As long as the L_{mn} remains less than 1, the system is stable and when this index exceeds the value 1,the system loses its stability and the voltage collapses.

Line voltage stability indices

3. Line Stability Factor (LQP)

- The line stability factor, LQP, based on the same concept as two previous line VSIs.

$$LQP = 4\left(\frac{X}{V_s^2}\right)\left(Q_r + \frac{P_s^2 X}{V_s^2}\right)$$

- For the transmission line to be stable, it should be LQP<1.
- In this index, the lines are assumed to be lossless (R/X<<1) and the shunt admittance of lines is neglected.
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Line voltage stability indices

4. Line Stability Index (L_p)

- The L_p has been designated based on the same concept as previous lineVSIs.
- For any value of L_p greater than one, the system is considered as unstable. The L_p for a transmission line is defined as follows:

$$L_p = \frac{4RP_r}{(V_s \ \cos(\theta - \delta))^2}$$

Line voltage stability indices

5. Voltage reactive power index($\boldsymbol{VQI_{Line}}$ **)**

- This index has been derived based on the same concept as L_p. This index is given by:

$$VQI_{Line} = \frac{4Q_r}{|B|V_s^2}$$

- The critical value(CV)of VQI_{Line} is 1 and, beyond this value, the voltage will collapse.
- In this index, δ is assumed to be zero and the line shunt admittance is neglected.

Bus voltage stability indices

Bus voltage stability indices

- Bus VSIs determine the voltage stability of system buses and do not provide any information about the weak facilities with potential voltage problems in the system.
- So, the bus voltage stability indices cannot be used for the determination of the weak facilities.

1. **Voltage collapse prediction index(*$VCPI_{bus}$*)**

- The $VCPI_{bus}$ is derived from the basic power flow equation and its value varies between 0 and 1.

Bus voltage stability indices

1. Voltage collapse prediction index($VCPI_{bus}$)

- If the value of $VCPI_{bus}$ reaches 1, the voltage at a bus has collapsed.
- The formulation of this index is as follows:

$$VCPI_{bus} = \min\{VCPI_i\}$$

- where

$$VCPI_i = \left| 1 - \frac{\sum_{\substack{m=1 \\ m \neq i}}^{N} V'_m}{V_i} \right|$$

Bus voltage stability indices

1. Voltage collapse prediction index($VCPI_{bus}$)

and

$$V'_m = \frac{Y_{im}}{\sum_{\substack{j=1 \\ j \neq i}}^{N} Y_{ij}} V_m$$

- where V_i and V_m are the voltage phasors at bus m and bus i, N is the number of buses, and Y_{im} is the admittance between the buses i and m.
- This index is based on the concept that the voltage equations must have a solution. In the matrix form, the determinant of a matrix must not be zero.

Bus voltage stability indices

2. L-index

- L-index based on the solution of the power flow equations. This index has been derived as follows:

$$L = \max_{j \in \alpha_L}\{L_j\} = \max_{j \in \alpha_L}\left|1 - \frac{\sum_{i \in \alpha_G} F_{ji}V_i}{V_j}\right|$$

- where a_L is the set of load buses, a_G is the set of generator buses, V_j and V_i are the voltage phasors at bus *j* and bus *i*, and F_{ji} is the element in *j-th* row and *i-th* column of matrix *F* whose elements are generated from the admittance matrix as

Bus voltage stability indices

2. L-index

$$F = -Y_{LL}^{-1} Y_{LG}$$

and

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix}$$

- The values of L-index vary between 0 (no load condition) and 1 (voltage collapse) and the concept of this index is the same as that of VCPI_{bus}.

Bus voltage stability indices

3. Voltage stability index (VSI_{bus})

- This index is based on the fact that in the vicinity of voltage collapse point an increase in the apparent power flow at the sending end of the line no longer yields an increase in the received power.
- This index is given by

$$VSI_i = \left[1 + \left(\frac{I_i}{V_i}\right)\left(\frac{\Delta V_i}{\Delta I_i}\right)\right]^{\alpha}$$

Overall voltage stability indices

Overall voltage stability indices

- This type of VSIs is not related to the system buses and lines.
- So, the overall VSI cannot determine the weakest bus or line and can only predict the system collapse point.

1. Network sensitivity approach (SG)

- SG has been proposed to calculate voltage stability and senses how far the system is from its collapse point.

Overall voltage stability indices

1. Network sensitivity approach (SG)

- SG contains tow indices as follows

$$SG_p = \frac{P_{gt}}{P_{dt}} \qquad SG_q = \frac{P_{gt}}{Q_{dt}}$$

- The system approaches to its collapse point when SGp and SGq increment gradually, causing a sharp rise to infinite values.
- The SG is based on the P-V curve and it assumed that the power system efficiency is constant.

Thank You