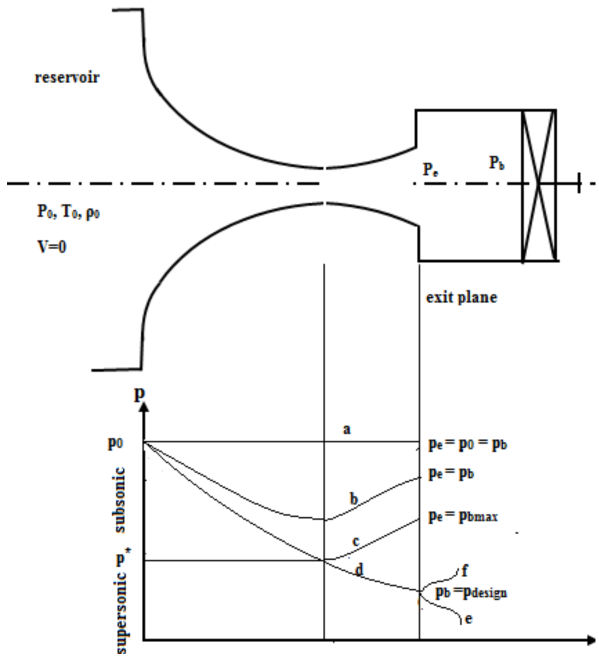
Sheet /Dr

Convergent Divergent Nozzle



(a) The pressure is the same throughout the nozzle $p_e = p_b = p_0$ (there is no flow).

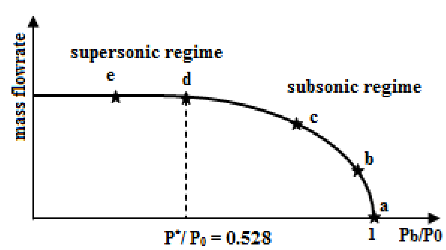
(b) If the back pressure slightly reduce, a mass flowrate passes the nozzle and the pressure decreases in the convergent portion and reaches its maximum velocity at the throat. Then the flow decelerate in the divergent portion and the flow totally subsonic and like the conventional venture. $p_e = p_b$

(c) More reduction in back pressure, and the mass flowrate accelerated such that the velocity decreasing while the rate of mass density of air increasing more till the flow is chocked at the throat. While the revers case in the divergent portion, the flow is maximum. $p_e = p_b = p_{b\ max}$ the pressure at which the flow is chocked at the throat and the flow is subsonic at the divergent portion of the nozzle.

(d) since the flow is chocked at the throat, any reduction in the back pressure makes the flow supersonic in the divergent section of the nozzle and the flow leaves the exit plane at pressure which is called 'Design pressure'. $p_b = p_{design} = p_e$

(e) If $p_b < p_{design} = p_e$ some expansion and compression waves "under expansion".

(f) When $p_b > p_{design} = p_e$ an oblique shock occurs outside the nozzle "over expansion".



Problem (1) isentropic flow of air in convergent nozzle is supplied from a large reservoir in which the pressure and temperature are 650 kpa and 350 k respectively. At a section in the nozzle where the area is 26 cm^2, the Mach is found to be 0.5, if the is nozzle discharged to a back pressure 270 kPa. Find the exit area.

ANS: 19.4 cm^2

Solution

Enter table with M = 0.5

$$p_1 / p_0 = 0.8430 \Rightarrow p_1 = 548 kPa$$

$$T_1 / T_0 = 0.9523 \Rightarrow T_1 = 333.5 k$$

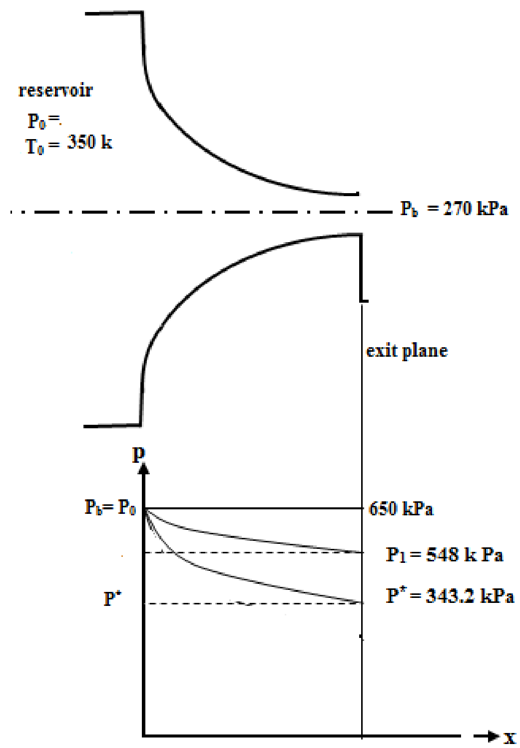
$$p^* / p_0 = 0.528 \Rightarrow p^* = 343.2 kPa$$

$$A / A^* = 1.339 \Rightarrow A^* = 19.4 cm^2$$

$$p_b / p_0 = 270 / 650 = 0.4155$$

$$enter\ table\ with\ it \Rightarrow M = 1.195$$

$$the\ nozzle\ is\ chocked\ \ A_e = A^* = 19.4 cm^2$$



Problem (2) Air flow isentropically through a converging nozzle attached to a large tank, where the pressure is 171 kPa and temperature is 27 C̊. At the inlet to the nozzle the Mach number is 0.2, the nozzle discharge to the atmosphere. The discharge area is 0.015m^2. Determine the magnitude and direction of the force that must be applied to hold the nozzle in place.

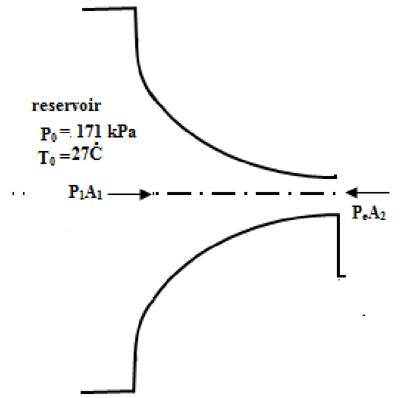
ANS: 1560N

Solution

$$p_1A_1 - p_eA_2 - p_b(A_1 - A_2) - R_x = \omega(v_2 - v_1)$$
$$p_e = p_b$$
$$p_1A_1 - p_b\,A_1 - R_x = \omega(v_2 - v_1)$$
$$R_x = (p_1 - p_b)A_1 - \omega(v_2 - v_1)$$

Enter table with M=0.2

$$p_1/p_0 = 0.9725 \Rightarrow p_1 = 171 \times 0.9725 = 166.3kPa$$
$$T_1/T_0 = 0.99206 \Rightarrow T_1 = 300 \times 0.99206 = 297.6k$$



$$\rho_1 = \frac{p_1}{RT_1} = \frac{166.3 \times 10^3}{287 \times 297.6} = 1.947kg/m^3$$
$$\frac{A_1}{A^*} = 2.9635$$
$$c_1 = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 297.6} = 345.8\,\text{m/s}$$
$$v_1 = M_1c_1 = 0.2 \times 345.8 = 69.159m/s$$
$$enter\ table\ with\ p_e/p_0 = 101.3/171 = 0.5923$$
$$M_e = 0.9\ ;T_e/T_0 = 0.8605\ ;T_e = 300 \times 0.8605 = 258.15k$$
$$A_2/A^* = 1.009 \Rightarrow A_1 = \frac{A_1/A^*}{A_2/A^*} \times A_2 = \frac{2.9635}{1.009} \times 0.015 = 0.04405m^2$$
$$v_2 = M_e \times \sqrt{kRT} = 0.9 \times \sqrt{1.4 \times 287 \times 258.15} = 289.8m/s$$
$$\omega_1 = \rho_1A_1v_1 = 1.947 \times 0.04405 \times 69.15 = 5.931kg/s$$
$$R_x = (166.3 - 101.3) \times 10^3 \times 0.04405 - 5.931 \times (289.8 - 69.159) = 1566\,\text{N}$$

Problem (3) Air flow isentropically through a converging nozzle discharges to atmosphere. At the section where the pressure is 179 kPa, the temperature is 39 C^o and the velocity is 177 m/s. determine the nozzle throat pressure.

ANS: 112 kPa

Solution

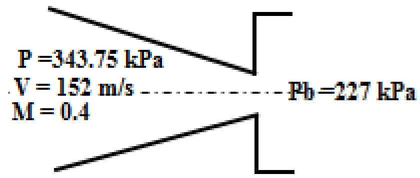
$$T_0 = T + \frac{v^2}{2c_p} = 312 + \frac{171^2}{2 \times 1005} = 327.2k$$
$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} \Rightarrow p_0 = 179 \times \left(\frac{327.6}{312}\right)^{3.5} = 212.3kPa$$
$$\frac{p^*}{p_0} = 0.528 \Rightarrow p^* = 0.528 \times 212.3 = 112kPa$$

Problem (4) Air flow isentropically through converging nozzle discharges into reservoir where the pressure is 227 kPa. If the pressure is 343.75 kPa and the speed is 152 m/s at the nozzle location where the Mach number is 0.4. Determine the pressure, speed and the Mach number at the nozzle exit plane.

ANS: 227 kPa, 323 m/s, 0.9

Solution

Enter isentropic table with M = 0.4



$$\frac{p}{p_0} = 0.8965 \Rightarrow p_0 = \frac{343.75}{0.8956} = 383.82kPa$$

$$\frac{T}{T_0} = 0.96899 \Rightarrow T_0 = \frac{T}{0.96899} = \frac{c^2 / kR}{0.96899} = \frac{(152/0.4)^2 / 1.4 \times 287}{0.96899} = 370.8k$$

$$\frac{p*}{p_0} = 0.528 \Rightarrow p* = 217.2kPa \text{ the nozzle not chocked}$$

$$\text{enter table with } \frac{p_e}{p_0} = \frac{227}{383.8} = 0.5914$$

$$M = 0.9; \quad \frac{T}{T_0} = 0.86058 \Rightarrow T = 370.8 \times 0.86058 = 319.1k$$

$$c = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 319.1} = 358.1 m/s$$

$$v = M \times c = 0.9 \times 358.1 = 322.2 m/s$$

Problem (5) Air flow through a converging- diverging nozzle isentropically, from a large tank where the pressure is 138kPa and temperature is 4 C^0. If the exit area is 1300 mm^2 and the back pressure is the atmospheric pressure (i.e. 101.3 kPa). Find the mass flowrate. ANS: 0.39 kg/s

Solution

$$\text{Enter table with } \frac{p_b}{p_0} = \frac{101.3}{138} = 0.7340$$

$$M = 0.68 \;\; ; \; \frac{T}{T_0} = 0.9153 \Rightarrow T = 0.9153 \times 277 = 253.5k$$

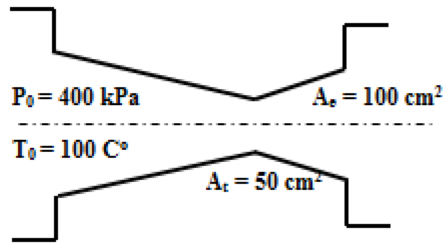
$$\omega = \rho v A = \frac{p}{RT} \times M \sqrt{kRT} \times A$$

$$\omega = \frac{101.3 \times 10^3}{287 \times 253.5} \times 0.68 \times \sqrt{1.4 \times 287 \times 253.5} \times 1300 \times 10^{-6} = 0.39 kg/s$$

Problem (6) A converging diverging nozzle is supplied from a large constant pressure air tank. The exit area of the nozzle is 100 cm^2 and the throat area is50 cm^2. The tank pressure and temperature are 400 kPa and 100 C°. find: (a) the maximum back pressure where the nozzle chocked, (b) find the mass flowrate at pressure of 200 and 300 kPa, and (c) determine the back pressure and exit Mach number for perfectly expanded flow in nozzle.

ANS: (a) 375.7 kPa (b) 4.1835 kg/s (c) 37.4 kPa 2.2

Solution



(a) $M_t = 1 \Rightarrow A_t = A^*$

$$Enter\ table\ with\ \frac{A_e}{A^*} = \frac{100}{50} = 2\ (subsonic\ region)$$

$$M_e = 0.3\ ;\ \frac{p_e}{p_0} = 0.9394 = \frac{p_{b\max}}{p_0}$$

$$p_{b\max} = 0.9394 \times 400 = 375.7 kPa$$

(b) since $p_b = 200$ and 300 kPa $< p_{bMax}$ *nozzle is chocked*

$$p^* = 0.528 \times 400 = 211.2 kPa$$

$$T^* = 0.8333 \times 373 = 310.7 k$$

$$\omega_e = \omega^* = \rho^* v^* A^* = \sqrt{\frac{k}{RT^*}} p^* A*$$

$$\omega^* = \sqrt{\frac{1.4}{287 \times 310.7}} \times 211.2 \times 10^3 \times 50^2 \times 10^{-4} = 4.18 kg/s$$

(c) Enter table with $\frac{A_e}{A^*} = 2$ (supersonic region)

$$\frac{p_b}{p_0} = \frac{p_{design}}{p_0} = 0.09352\ ;\ M = 2.2$$

$$p_{design} = 0.09352 \times 400 = 37.4 kPa$$

Problem (7) Air flow isentropically through a converging diverging nozzle attached to a large tank in which the pressure is 690 kPa and temperature is 278 k. the nozzle is operating at a design pressure of 101 .3 kPa. The exit area is 25.8 cm^2. Calculate the corresponding mass flowrate. If the temperature of air in the tank is increased to 1100 k (all pressure remaining the same), how will be the flow rate be affected?

Solution

$$p^* = 0.528 \times 690 = 364.3 kPa$$

enter table with

$$\frac{p_e}{p_0} = \frac{101.3}{690} = 0.1468 \text{(supersonic region)}$$

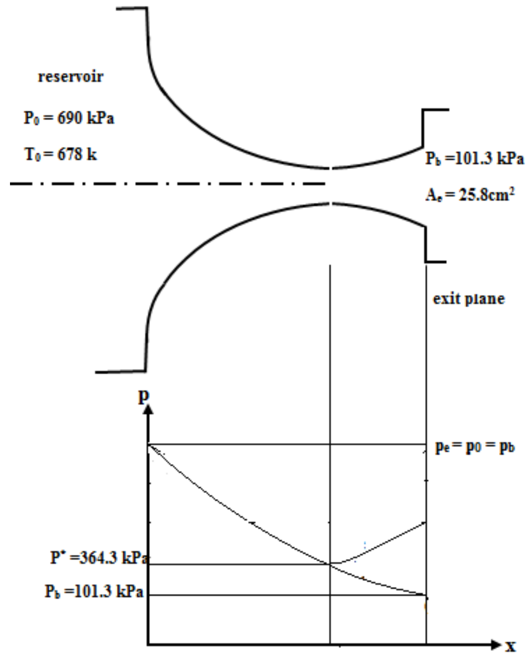
$$M_e = 1.91, \ \frac{T_e}{T_0} = 0.5781$$

$$T_e = 0.5781 \times 278 = 160.725$$

$$\omega_e = \rho_e v_e A_e = \sqrt{\frac{k}{RT_e}} P_e A_e M_e$$

$$\omega_e = \sqrt{\frac{1.4}{287 \times 160.725}} \times 101.3 \times 10^3 \times 25.8 \times 10^{-4} \times$$

$$\times 1.91 = 2.75 kg/s$$



$$when T_0 = 1100 k \ then$$

$$T_e = 0.528 \times 1100 = 580.8 k$$

$$\omega_e = \rho_e v_e A_e = \sqrt{\frac{k}{RT_e}} p_e A_e M_e = \sqrt{\frac{1.4}{287 \times 580.8}} \times 101.3 \times 10^3 \times 25.8 \times 10^{-4} \times 1.91 = 1.4 \text{ kg/s}$$

Problem (8) Nitrogen at pressure and temperature of 371 kPa and 400 k, enters a converging diverging nozzle with negligible velocity. The exhaust jet is directed against a large plate that perpendicular the jet axis. The flow leaves the nozzle at atmospheric pressure. The exit area is 0.003 m^2. Find the force required to hold the plate. (For nitrogen take the molecular mass 28 and k=1.4). ANS 650 N

Solution

$$p^* = 0.528 \times 371 = 195.88 kPa > 101.3 kPa \; nozzle \; is \; chocked$$

$$enter \; table \; with \; \frac{p_e}{p_0} = \frac{101.3}{371} = 0.273$$

$$M_e = 1.5 \qquad \frac{T_e}{T_0} = 0.6896 \Rightarrow T_e = 0.6896 \times 400 = 275.84k$$

$$c_e = \sqrt{kRT} = \sqrt{1.4 \times \frac{8314}{28} \times 275.84} = 338.6 m/s$$

$$v_e = M_e c_e = 1.5 \times 338.6 = 507.9 m/s$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{101.3 \times 1000}{296.9 \times 275.8} = 1.23 kg/m^3$$

$$\omega_e = \rho_e v_e A_e = 1.23 \times 507.9 \times 0.003 = 1.88 kg/s$$

$$R_x = \omega_e v_e = 1.88 \times 507.9 = 957.4N$$

Problem (9) Air expand from a large tank where the temperature and pressure are 500 k and 600 kPa respectively. The Mach number at the inlet of the nozzle is 0.6 and cross-sectional area is 7.2 cm^2. The exit area is 12 cm^2. Find the exit pressure and temperature when the nozzle operated under: (a) Maximum pressure, (b) design pressure.

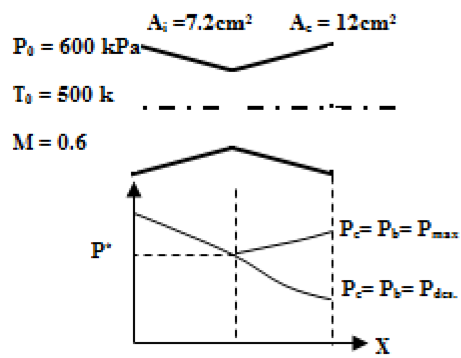
Solution

Enter isentropic table with M = 0.6

$$\frac{A_i}{A^*} = 1.1882$$

$$\frac{A_e}{A^*} = \frac{A_e}{A_i} \times \frac{A_i}{A^*} = \frac{12}{7.2} \times 1.1882 = 1.980$$

$$Enter \; isentropic \; table \; with \; \frac{A_e}{A^*} \; subsonic \; portion$$



$$M_e = 0.31, \quad \frac{T_e}{T_0} = 0.98114, \quad \frac{P_e}{P_0} = 0.9355$$

$$T_e = 500 \times 0.98114 = 490.5k$$

$$P_e = 600 \times 0.9355 = 561.32kPa$$

$$Enter \; isentropic \; table \; with \; \frac{A_e}{A^*} \; \text{supersonic} \; portion$$

$$M_e = 2.19, \quad \frac{P_e}{P_0} = 0.0955, \quad \frac{T_e}{T_0} = 0.5104$$

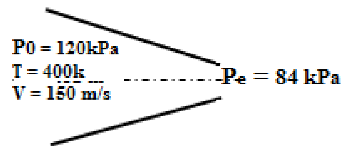
$P_e = P_{des.} = 0.0955 \times 600 = 57.3 kPa$

$T_e = 0.5104 \times 500 = 255k$

Problem (10) Air enters a converging adiabatic nozzle at 400k with a velocity of 150 m/s. If the stagnation pressure is 120 kPa, and the air exhausts to ambient where the pressure is 84 kPa. Find the mass flowrate?

Solution

$$\frac{p_e}{p_0} = \frac{84}{120} = 0.7 > 0.528 \quad \textit{the nozzle not chocked}$$



$$\textit{Enter table with } \frac{p_e}{p_0} = 0.7$$

$$M_e = 0.74, \qquad \frac{T_e}{T_0} = 0.90129$$

$$M_i = \frac{v_i}{c_i} = \frac{v_i}{\sqrt{kRT_i}} = \frac{150}{\sqrt{1.4 \times 287 \times 400}} = 0.374$$

$$\textit{Enter isentropic table with } M_i = 0.374$$

$$\frac{T_i}{T_0} = 0.97335 \Rightarrow T_0 = \frac{400}{0.97335} = 411k$$

$$\therefore T_e = 411 \times 0.90129 = 370.4k$$

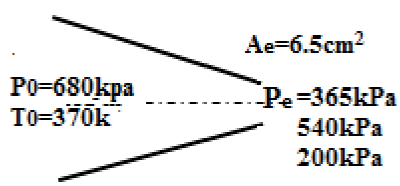
$$\frac{\overset{o}{m}}{A} = \sqrt{\frac{k}{RT_e}} p_e M_e = \sqrt{\frac{1.4}{287 \times 370.4}} \times 84 \times 10^3 \times 0.74 = 225.5 \frac{kg}{m^2 s}$$

Problem (11) A convergent has an exit area of 6.5 cm^2. Air enters the nozzle at stagnation pressure and temperature of 680 kPa and 370 k respectively. If the flow is isentropic, determine the mass flowrate of the flow for back pressure of:

(a) 359 kPa (b) 540 kPa (c) 200 kPa

Solution

$$\text{(a)}: p_b = 365 kPa$$



$$\frac{p_b}{p_0} = \frac{365}{680} = 0.5367 > 0.528 \ \textit{nozzle not chocked}$$

$$\textit{enter isentropic table with } p_b / p_0 = 0.5367$$

$$M_e = 0.99 \qquad \frac{T_e}{T_0} = 0.83611$$

$$T_e = 0.83611 \times 370 = 309.36k$$

$$\overset{o}{m} = \sqrt{\frac{k}{RT_e}} P_e A_e M_e = \sqrt{\frac{1.4}{287 \times 309.3}} \times 365 \times 10^3 \times 6.5 \times 10^{-4} \times 0.99 = 0.923 kg/s$$

(b) $p_b = 540 kPa$

$$\frac{p_b}{p_0} = \frac{540}{680} = 0.794 > 0.528 \quad \textit{the nozzle not chocked}$$

Enter isentropic table with $\frac{p_b}{p_0} = 0.794$

$$M_e = 0.58 \qquad \frac{T_e}{T_0} = 0.93696 \Rightarrow T_e = 0.93696 \times 370 = 346.7k$$

$$\overset{o}{m} = \sqrt{\frac{1.4}{287 \times 346.7}} \times 540 \times 10^3 \times 6.5 \times 10^{-4} \times 0.58 = 0.0028 kg/s$$

(c) $p_b = 200 kPa$

$$\frac{p_b}{p_0} = \frac{200}{680} = 0.249 < 0.528 \quad \textit{the nozzle is chocked}$$

$$M_e = M^* = 1 \qquad \frac{p_e = p^*}{p_0} = 0.528 \Rightarrow p_e = p^* = 0.528 \times 680 = 359 kPa$$

$$\frac{T_e = T^*}{T_0} = 0.8333 \Rightarrow T_e = T^* = 0.8333 \times 370 = 308.3k$$

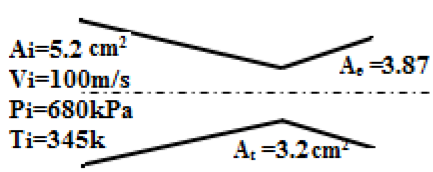
$$\overset{o}{m} = \sqrt{\frac{k}{RT^*}} P^* A^* M^* = \sqrt{\frac{1.4}{287 \times 308.3}} \times 359 \times 6.5 \times 10^{-1} \times 1 = 0.928 kg/s$$

<u>Problem (12)</u> Air flow isentropically through a convergent divergent nozzle with inlet area of 5.2 cm^2, minimum area is 3.2 cm^2 and exit area of 3.87 cm^2. At inlet the air velocity 100 m/s, pressure 680 kPa, and temperature 345 k. Determine:

a- The mass flowrate through the nozzle.

b- The Mach number of the minimum area section.

c- The velocity and pressure at the exit section.



$\overset{o}{m}$?

$$c_i = \sqrt{kRT_i} = \sqrt{1.4 \times 287 \times 345} = 372.3 m/s$$

$$M_i = \frac{v_i}{c_i} = \frac{100}{372.3} = 0.2685$$

$$\overset{o}{m} = \sqrt{\frac{k}{RT_i}} P_i A_i M_i = \sqrt{\frac{1.4}{287 \times 345}} \times 680 \times 10^3 \times 5.2 \times 10^{-4} \times 0.2685 = 0.357 \, \text{kg/ s}$$

b– M_t?

$$\textit{Enter isentropic table with } M_i = 0.2685$$

$$\frac{p_i}{p_0} = 0.9506, \qquad \frac{T_i}{T_0} = 0.98563, \qquad \frac{A_i}{A^*} = 2.23$$

$$p_0 = \frac{680}{0.9506} = 715.3 kPa, \qquad T_0 = \frac{345}{0.98563} = 350k$$

$$\frac{A_t}{A^*} = \frac{A_t}{A_i} \times \frac{A_i}{A^*} = \frac{3.2}{5.2} \times 2.23 = 1.372$$

$$\textit{Enter isentropic table with } \frac{A_t}{A^*} = 1.372$$

$$M_t = 0.48$$

$$c - v_e, p_e, T_e \,?$$

$$\frac{A_e}{A^*} = \frac{A_e}{A_i} \times \frac{A_i}{A^*} = \frac{3.87}{5.2} \times 2.23 = 1.66$$

$$\textit{Enter isentropic table with } \frac{A_e}{A^*} = 1.66$$

$$M_e = 0.38, \qquad \frac{T_e}{T_0} = 0.97193, \qquad \frac{p_e}{p_0} = 0.9051$$

$$T_e = 350 \times 0.97193 = 340.17k$$

$$p_e = 715.3 \times 0.9051 = 647.4 kPa$$

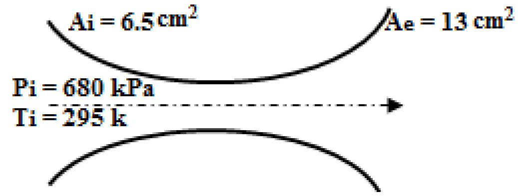
$$c_e = \sqrt{kRT_e} = \sqrt{1.4 \times 287 \times 340.17} = 369.7 m/s$$

$$v_e = M_e c_e = 0.38 \times 369.7 = 140.5 m/s$$

Problem (13) Air flow isentropically through a supersonic convergent divergent nozzle with 5 kg/s. At the inlet the pressure is 680 kPa and the temperature is 295 k, and the area is 6.5 cm^2. If the exit area is 13 cm^2 calculate:

a- The stagnation pressure and temperature.
b- The exit Mach number.
c- The exit pressure and temperature.
d- The area and velocity at the throat.
e- What will be the maximum rate of flow and corresponding exit Mach number if the flow completely subsonic in the nozzle?

$a-\quad P_0\,?\ T_0\,?$



$$\overset{o}{m} = \rho v A = \frac{p_i}{RT_i} M_i \sqrt{kRT_i}\, A_i$$

$$M_i = \frac{\overset{o}{m}\, RT_i}{p_i \sqrt{kRT_i}\, A_i} = \frac{\overset{o}{m}\sqrt{RT_i}}{p_i A_i \sqrt{k}} = \frac{0.5\times\sqrt{287\times295}}{680\times10^3\times6.5\times10^{-4}\times\sqrt{1.4}} = 0.278$$

$Enter\ isentropic\ table\ with\ M_i = 0.278$

$$\frac{p_i}{p_0} = 0.9470 \Rightarrow p_0 = \frac{680}{0.9470} = 718kPa$$

$$\frac{T_i}{T_0} = 0.9845 \Rightarrow T_0 = \frac{295}{0.9845} = 299.6k$$

$$\frac{A_i}{A^*} = 2.16$$

$b - M_e\,?$

$$\frac{A_e}{A^*} = \frac{A_i}{A^*}\times\frac{A_e}{A_i} = 2.16\times\frac{13}{6.5} = 4.32$$

$Enter\ isentropic\ table\ with\ \frac{A_e}{A^*} = 4.32$

$$M_e = 3.0, \quad \frac{p_e}{p_0} = 0.027, \quad \frac{T_e}{T_0} = 0.337$$

$c - p_e\,?\quad T_e\,?$

$$p_e = 718\times0.027 = 19.3kPa$$

$$T_e = 299.6\times0.337 = 106.9k$$

$d - A^*\,?\ \ v^*\,?$

$$\frac{A_i}{A^*} = 2.16 \Rightarrow A^* = \frac{6.5}{2.16} = 3cm^2$$

$$\frac{T^*}{T_0} = 0.8333 \Rightarrow T^* = 0.8333\times299.6 = 249.6k$$

$$v^* = c^* = \sqrt{kRT^*} = \sqrt{1.4\times287\times249.6} = 316.7m/s$$

$e - M \ ?\overset{o}{m}_{\max}?$

$$\frac{p^*}{p_0} = 0.528 \Rightarrow p^* = 0.528 \times 718 = 379 kPa$$

$$\overset{o}{m}_{\max} = \sqrt{\frac{k}{RT^*}} P^* M^* A^* = \sqrt{\frac{1.4}{287 \times 249.6}} \times 379 \times 10^3 \times 1 \times 3 \times 10^{-4} = 0.502 kg/s$$

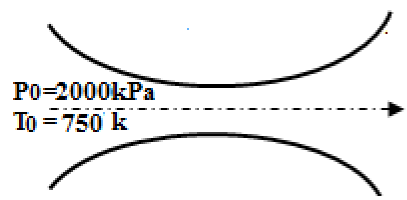
$$Enter\ isentropic\ table\ with\ \frac{A_e}{A^*} = 4.32 (\text{subsonic region})$$

$$\mathrm{M} = 0.13$$

<u>Problem (14)</u> Air at stagnation condition of 2 Mpa and 750 k flow isentropically through a convergent divergent nozzle. If the maximum flowrate is 5.4 kg/s, determine:

a- the throat area.

b- The velocity, pressure and temperature at the nozzle if the exit area is three time as large as the throat area.



$$a - \text{since}\ \overset{o}{m}_{\max}\ it\ mean\ that\ M_t = 1$$

$$\frac{p^*}{p_0} = 0.528 \Rightarrow p^* = 0.528 \times 2000 = 1056 kPa$$

$$\frac{T^*}{T_0} = 0.8333 \Rightarrow T^* = 0.8333 \times 750 = 625 k$$

$$\overset{o}{m}_{\max} = \sqrt{\frac{k}{RT^*}} P^* A^* M^* \Rightarrow A^* = \frac{\overset{o}{m}_{\max} \sqrt{RT^*}}{\sqrt{k} P^* M^*}$$

$$A^* = \frac{5.4 \times \sqrt{287 \times 625}}{\sqrt{1.4} \times 1056 \times 10^3 \times 1} = 18.3 cm^2$$

$$b - Enter\ isentropic\ table\ with\ \frac{A_e}{A^*} = 3, \quad M_e = 2.64$$

$$\frac{p_e}{p_0} = 0.04711 \Rightarrow p_e = 0.04711 \times 2000 = 94.22 kPa$$

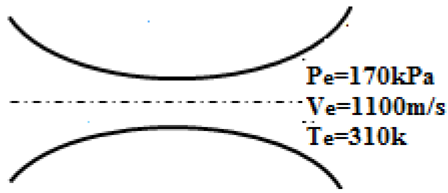
$$\frac{T_e}{T_0} = 0.4177 \Rightarrow T_e = 0.4177 \times 750 = 313.3 k$$

$$v_e = M_e \sqrt{kRT_e} = 2.64 \times \sqrt{1.4 \times 287 \times 313.3} = 936.2 m/s$$

Problem (15) Find the throat and the exit area for a critical flow nozzle handling air at the rate of 6.7 kg/s when the desired exit velocity is 1100 m/s with the stream pressure is 170 kPa and temperature of 310 k.

$$M_e = \frac{v_e}{c_e} = \frac{v_e}{\sqrt{kRT_e}} = \frac{1100}{\sqrt{1.4 \times 287 \times 310}} = 3.1$$

$$Enter\ isentropic\ table\ with M_e = 3.1$$



$$\frac{A_e}{A^*} = 4.65, \qquad \frac{p_e}{p_0} = 0.023, \qquad \frac{T_e}{T_0} = 0.342$$

$$p_0 = \frac{170}{0.023} = 7391.3\text{kPa}, \qquad T_0 = \frac{310}{0.342} = 906.4$$

$$p^* = 0.528 \times 7391.3 = 3902.6kPa$$

$$T^* = 0.8333 \times 906.4 = 755.3k$$

$$if\ \overset{o}{m} = \overset{o}{m}_{\max} = \sqrt{\frac{k}{RT^*}} P^* M^* A^*$$

$$A^* = \frac{\overset{o}{m}}{\sqrt{k}\, P^* M^*} \sqrt{RT^*} = \frac{6.7 \times \sqrt{287 \times 755.3}}{\sqrt{1.4} \times 3902.6 \times 10^3 \times 1} = 6.75 cm^2$$

$$\frac{A_e}{A^*} = 4.65 \Rightarrow A_e = 4.65 \times 6.75 = 31.38 cm^2$$

$$f\ \overset{o}{m} \neq \overset{o}{m}_{\max} \Rightarrow A_e = \frac{\overset{o}{m}}{\sqrt{k}\, P_e M_e} \sqrt{RT_e} = \frac{6.7 \times \sqrt{287 \times 310}}{\sqrt{1.4} \times 170 \times 10^3 \times 3.1} = 33.1 cm^2$$

Problem (16) In laboratory test section, the stagnation speed of sound in air flow is measured to be 450 m/s. At some downstream location the sound velocity is also measured to be 316 m/s, find the flow Mach number at this location.

$$\frac{T_0}{T_1} = 1 + \frac{k-1}{2} M_1^2 \qquad \times \frac{kR}{kR}$$

$$\frac{kR}{kR} \frac{T_0}{T_1} = 1 + \frac{k-1}{2} M_1^2 = \frac{c_0^2}{c_1^2}$$

$$\frac{450^2}{316^2} - 1 = 0.2 M_1^2 \Rightarrow M_1 = 2.267$$

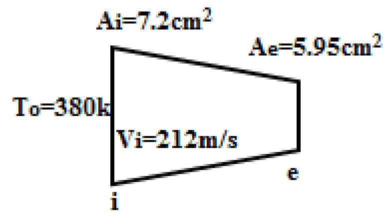
OR

$$T_0 = \frac{c_0^2}{kR} = \frac{450^2}{1.4\times 287} = 502.98k$$

$$T_1 = \frac{c_1^2}{kR} = \frac{316^2}{1.4\times 287} = 248.52k$$

$$\text{Enter isentropic table with } \frac{T_1}{T_0} = \frac{248.52}{502.98} = 0.493 \Rightarrow M_1 = 2.267$$

<u>Problem (17)</u> Air flows from large reservoir where the temperature is 380 k, and enters a convergent nozzle. At inlet plane of the nozzle the velocity is 212 m/s, and area is 7.2 cm^2. If the exit plane area is 5.95 cm^2, determine the exit Mach number and the exit temperature.



$$T_0 = T_i + \frac{v_i^2}{2c_p} \Rightarrow T_i = 380 - \frac{212^2}{2\times 1005} = 357.63k$$

$$c_i = \sqrt{kRT_i} = \sqrt{1.4\times 287\times 357.63} = 379.07m/s$$

$$M_i = \frac{v_i}{c_i} = \frac{212}{379.07} = 0.559$$

$$\text{Enter isentropic table with } M_i = 0.559 \Rightarrow \frac{A_i}{A^*} = 1.240$$

$$\frac{A_e}{A^*} = \frac{A_e}{A_i}\times\frac{A_i}{A^*} = \frac{5.95}{7.2}\times 1.240 = 1.0247$$

$$\text{Enter isentropic table with } \frac{A_e}{A^*} = 1.0247$$

$$M_e = 0.84, \quad \frac{T_e}{T_0} = 0.8763 \Rightarrow T_e = 380\times 0.8763 = 333k$$

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