11 Rolling-Contact Bearings

Chapter Outline

The terms *rolling-contact bearing, antifriction bearing,* and *rolling bearing* are all used to describe that class of bearing in which the main load is transferred through elements in rolling contact rather than in sliding contact. In a rolling bearing the starting friction is about twice the running friction, but still it is negligible in comparison with the starting friction of a sleeve bearing. Load, speed, and the operating viscosity of the lubricant do affect the frictional characteristics of a rolling bearing. It is probably a mistake to describe a rolling bearing as "antifriction," but the term is used generally throughout the industry.

From the mechanical designer's standpoint, the study of antifriction bearings differs in several respects when compared with the study of other topics because the bearings they specify have already been designed. The specialist in antifriction-bearing design is confronted with the problem of designing a group of elements that compose a rolling bearing: these elements must be designed to fit into a space whose dimensions are specified; they must be designed to receive a load having certain characteristics; and finally, these elements must be designed to have a satisfactory life when operated under the specified conditions. Bearing specialists must therefore consider such matters as fatigue loading, friction, heat, corrosion resistance, kinematic problems, material properties, lubrication, machining tolerances, assembly, use, and cost. From a consideration of all these factors, bearing specialists arrive at a compromise that, in their judgment, is a good solution to the problem as stated.

We begin with an overview of bearing types; then we note that bearing life cannot be described in deterministic form. We introduce the invariant, the statistical distribution of life, which is strongly Weibullian.[1] There are some useful deterministic equations addressing load versus life at constant reliability, and we introduce the catalog rating at rating life.

The reliability-life relationship involves Weibullian statistics. The load-life-reliability relationship, combines statistical and deterministic relationships giving the designer a way to move from the desired load and life to the catalog rating in *one* equation.

Ball bearings also resist thrust, and a unit of thrust does different damage per revolution than a unit of radial load, so we must find the equivalent pure radial load that does the same damage as the existing radial and thrust loads. Next, variable loading, stepwise and continuous, is approached, and the equivalent pure radial load doing the same damage is quantified. Oscillatory loading is mentioned.

With this preparation we have the tools to consider the selection of ball and cylindrical roller bearings. The question of misalignment is quantitatively approached.

Tapered roller bearings have some complications, and our experience so far contributes to understanding them.

Having the tools to find the proper catalog ratings, we make decisions (selections), we perform a design assessment, and the bearing reliability is quantified. Lubrication and mounting conclude our introduction. Vendors' manuals should be consulted for specific details relating to bearings of their manufacture.

11–1 Bearing Types

Bearings are manufactured to take pure radial loads, pure thrust loads, or a combination of the two kinds of loads. The nomenclature of a ball bearing is illustrated in Fig. 11–1, which also shows the four essential parts of a bearing. These are the outer ring, the inner ring, the balls or rolling elements, and the separator. In low-priced bearings, the

[1]To completely understand the statistical elements of this chapter, the reader is urged to review Chap. 20, Secs. 20–1 through 20–3.

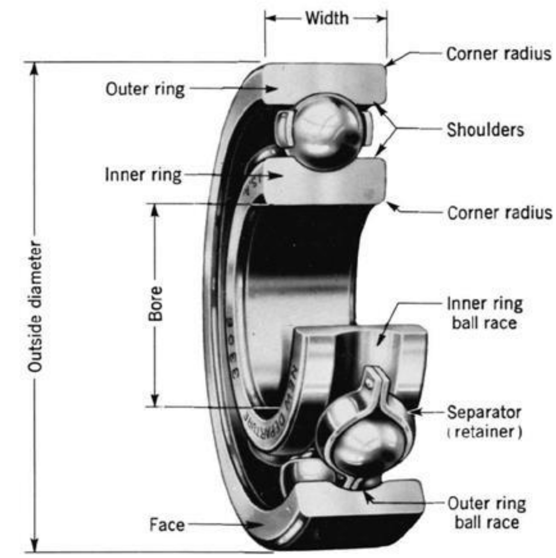


Figure 11–1

Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*

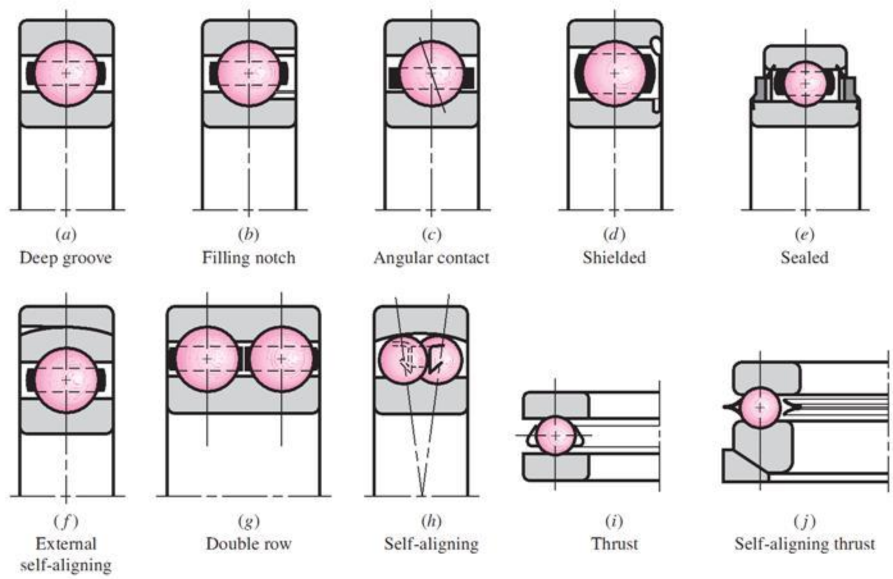


Figure 11–2

Various types of ball bearings.

separator is sometimes omitted, but it has the important function of separating the elements so that rubbing contact will not occur.

In this section we include a selection from the many types of standardized bearings that are manufactured. Most bearing manufacturers provide engineering manuals and brochures containing lavish descriptions of the various types available. In the small space available here, only a meager outline of some of the most common types can be given. So you should include a survey of bearing manufacturers' literature in your studies of this section.

Some of the various types of standardized bearings that are manufactured are shown in Fig. 11–2. The single-row deep-groove bearing will take radial load as well as some thrust load. The balls are inserted into the grooves by moving the inner ring to an eccentric position. The balls are separated after loading, and the separator is then inserted. The use of a filling notch (Fig. 11–2*b*) in the inner and outer rings enables a greater number of balls to be inserted, thus increasing the load capacity. The thrust capacity is decreased, however, because of the bumping of the balls against the edge of the notch when thrust loads are present. The angular-contact bearing (Fig. 11–2*c*) provides a greater thrust capacity.

All these bearings may be obtained with shields on one or both sides. The shields are not a complete closure but do offer a measure of protection against dirt. A variety of bearings are manufactured with seals on one or both sides. When the seals are on both sides, the bearings are lubricated at the factory. Although a sealed bearing is supposed to be lubricated for life, a method of relubrication is sometimes provided.

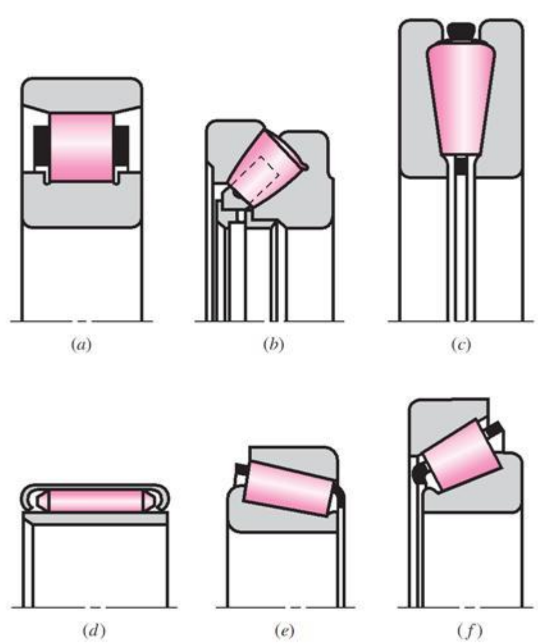
Single-row bearings will withstand a small amount of shaft misalignment of deflection, but where this is severe, self-aligning bearings may be used. Double-row bearings are made in a variety of types and sizes to carry heavier radial and thrust loads. Sometimes two single-row bearings are used together for the same reason, although a double-row bearing will generally require fewer parts and occupy less space. The one-way ball thrust bearings (Fig. 11–2*i*) are made in many types and sizes.

Some of the large variety of standard roller bearings available are illustrated in Fig. 11–3. Straight roller bearings (Fig. 11–3*a*) will carry a greater radial load than ball bearings of the same size because of the greater contact area. However, they have the disadvantage of requiring almost perfect geometry of the raceways and rollers. A slight misalignment will cause the rollers to skew and get out of line. For this reason, the retainer must be heavy. Straight roller bearings will not, of course, take thrust loads.

Helical rollers are made by winding rectangular material into rollers, after which they are hardened and ground. Because of the inherent flexibility, they will take considerable misalignment. If necessary, the shaft and housing can be used for raceways instead of separate inner and outer races. This is especially important if radial space is limited.

Figure 11–3

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*



The spherical-roller thrust bearing (Fig. 11–3*b*) is useful where heavy loads and misalignment occur. The spherical elements have the advantage of increasing their contact area as the load is increased.

Needle bearings (Fig. 11–3*d*) are very useful where radial space is limited. They have a high load capacity when separators are used, but may be obtained without separators. They are furnished both with and without races.

Tapered roller bearings (Fig. 11–3*e*, *f*) combine the advantages of ball and straight roller bearings, since they can take either radial or thrust loads or any combination of the two, and in addition, they have the high load-carrying capacity of straight roller bearings. The tapered roller bearing is designed so that all elements in the roller surface and the raceways intersect at a common point on the bearing axis.

The bearings described here represent only a small portion of the many available for selection. Many special-purpose bearings are manufactured, and bearings are also made for particular classes of machinery. Typical of these are:

- Instrument bearings, which are high-precision and are available in stainless steel and high-temperature materials
- Nonprecision bearings, usually made with no separator and sometimes having split or stamped sheet-metal races
- Ball bushings, which permit either rotation or sliding motion or both
- Bearings with flexible rollers

11–2 Bearing Life

When the ball or roller of rolling-contact bearings rolls, contact stresses occur on the inner ring, the rolling element, and on the outer ring. Because the curvature of the contacting elements in the axial direction is different from that in the radial direction, the equations for these stresses are more involved than in the Hertz equations presented in Chapter 3. If a bearing is clean and properly lubricated, is mounted and sealed against the entrance of dust and dirt, is maintained in this condition, and is operated at reasonable temperatures, then metal fatigue will be the only cause of failure. Inasmuch as metal fatigue implies many millions of stress applications successfully endured, we need a quantitative life measure. Common life measures are

- Number of revolutions of the inner ring (outer ring stationary) until the first tangible evidence of fatigue
- Number of hours of use at a standard angular speed until the first tangible evidence of fatigue

The commonly used term is *bearing life,* which is applied to either of the measures just mentioned. It is important to realize, as in all fatigue, life as defined above is a stochastic variable and, as such, has both a distribution and associated statistical parameters. The life measure of an individual bearing is defined as the total number of revolutions (or hours at a constant speed) of bearing operation until the failure criterion is developed. Under ideal conditions, the fatigue failure consists of spalling of the load-carrying surfaces. The American Bearing Manufacturers Association (ABMA) standard states that the failure criterion is the first evidence of fatigue. The fatigue criterion used by the Timken Company laboratories is the spalling or pitting of an area of 0.01 in^2. Timken also observes that the useful life of the bearing may extend considerably beyond this point. This is an operational definition of fatigue failure in rolling bearings.

The *rating life* is a term sanctioned by the ABMA and used by most manufacturers. The rating life of a group of nominally identical ball or roller bearings is defined as the number of revolutions (or hours at a constant speed) that 90 percent of a group of bearings will achieve or exceed before the failure criterion develops. The terms *minimum life, L_{10} life,* and *B_{10} life* are also used as synonyms for rating life. The rating life is the 10th percentile location of the bearing group's revolutions-to-failure distribution.

Median life is the 50th percentile life of a group of bearings. The term *average life* has been used as a synonym for median life, contributing to confusion. When many groups of bearings are tested, the median life is between 4 and 5 times the L_{10} life.

11–3 Bearing Load Life at Rated Reliability

When nominally identical groups are tested to the life-failure criterion at different loads, the data are plotted on a graph as depicted in Fig. 11–4 using a log-log transformation. To establish a single point, load F_1 and the rating life of group one $(L_{10})_1$ are the coordinates that are logarithmically transformed. The reliability associated with this point, and all other points, is 0.90. Thus we gain a glimpse of the load-life function at 0.90 reliability. Using a regression equation of the form

$$FL^{1/a} = \text{constant} \tag{11–1}$$

the result of many tests for various kinds of bearings result in

- $a = 3$ for ball bearings
- $a = 10/3$ for roller bearings (cylindrical and tapered roller)

A bearing manufacturer may choose a rated cycle value of 10^6 revolutions (or in the case of the Timken Company, $90(10^6)$ revolutions) or otherwise, as declared in the manufacturer's catalog to correspond to a basic load rating in the catalog for each bearing manufactured, as their rating life. We shall call this the *catalog load rating* and display it algebraically as C_{10}, to denote it as the 10th percentile rating life for a particular bearing in the catalog. From Eq. (11–1) we can write

$$F_1 L_1^{1/a} = F_2 L_2^{1/a} \tag{11–2}$$

and associate load F_1 with C_{10}, life measure L_1 with L_{10}, and write

$$C_{10} L_{10}^{1/a} = F L^{1/a}$$

where the units of L are revolutions.

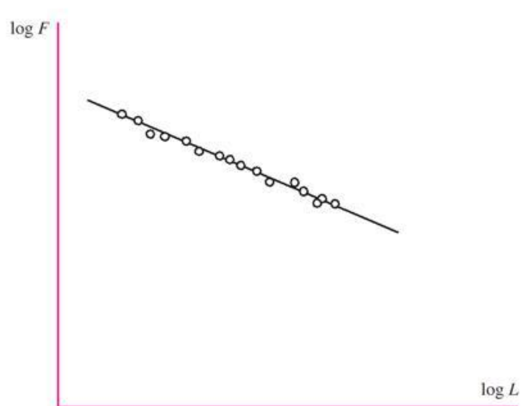


Figure 11–4

Typical bearing load-life log-log curve.

Further, we can write

$$C_{10}(L_R n_R 60)^{1/a} = F_D(L_D n_D 60)^{1/a}$$

- C_{10}: catalog rating, lbf or kN
- L_R: rating life in hours
- n_R: rating speed, rev/min
- F_D: desired radial load, lbf or kN
- L_D: desired life, hours
- n_D: desired speed, rev/min

Solving for C_{10} gives

$$C_{10} = F_D\left(\frac{L_D n_D 60}{L_R n_R 60}\right)^{1/a} \tag{11–3}$$

EXAMPLE 11–1

Consider SKF, which rates its bearings for 1 million revolutions, so that L_{10} life is $60L_R n_R = 10^6$ revolutions. The $L_R n_R 60$ product produces a familiar number. Timken, for example, uses $90(10^6)$ revolutions. If you desire a life of 5000 h at 1725 rev/min with a load of 400 lbf with a reliability of 90 percent, for which catalog rating would you search in an SKF catalog?

Solution

From Eq. (11–3),

$$C_{10} = F_D\left(\frac{L_D n_D 60}{L_R n_R 60}\right)^{1/a} = 400\left[\frac{5000(1725)60}{10^6}\right]^{1/3} = 3211\ \text{lbf} = 14.3\ \text{kN}$$

If a bearing manufacturer rates bearings at 500 h at $33\frac{1}{3}$ rev/min with a reliability of 0.90, then $L_R n_R 60 = 500(33\frac{1}{3})60 = 10^6$ revolutions. The tendency is to substitute 10^6 for $L_R n_R 60$ in Eq. (11–3). Although it is true that the 60 terms in Eq. (11–3) as displayed cancel algebraically, they are worth keeping, because at some point in your keystroke sequence on your hand-held calculator the manufacturer's magic number (10^6 or some other number) will appear to remind you of what the rating basis is and those manufacturers' catalogs to which you are limited. Of course, if you evaluate the bracketed quantity in Eq. (11–3) by alternating between numerator and denominator entries, the magic number will not appear and you will have lost an opportunity to check.

11–4 Bearing Survival: Reliability versus Life

At constant load, the life measure distribution is right skewed as depicted in Fig. 11–5. Candidates for a distributional curve fit include lognormal and Weibull. The Weibull is by far the most popular, largely because of its ability to adjust to varying amounts of skewness. If the life measure is expressed in dimensionless form as $x = L/L_{10}$, then the reliability can be expressed as [see Eq. (20–24), p. 970]

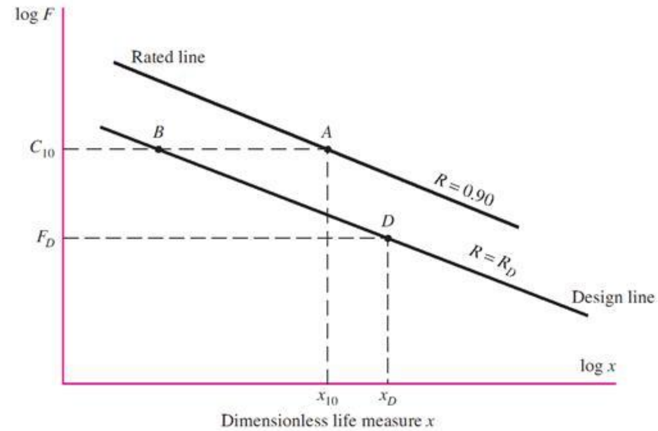
$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] \tag{11–4}$$

where R = reliability

x = life measure dimensionless variate, L/L_{10}

Figure 11–5

Constant reliability contours. Point A represents the catalog rating C_{10} at $x = L/L_{10} = 1$. Point B is on the target reliability design line R_D, with a load of C_{10}. Point D is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load F_D.



θ = characteristic parameter corresponding to the 63.2121 percentile value of the variate

b = shape parameter that controls the skewness

Because there are three distributional parameters, x_0, θ, and b, the Weibull has a robust ability to conform to a data string. Also, in Eq. (11–4) an explicit expression for the cumulative distribution function is possible:

$$F = 1 - R = 1 - \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] \tag{11–5}$$

EXAMPLE 11–2

Construct the distributional properties of a 02-30 mm deep-groove ball bearing if the Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$. Find the mean, median, 10th percentile life, standard deviation, and coefficient of variation.

Solution

From Eq. (20–28), p. 971, the mean dimensionless life μ_x is

Answer

$$\mu_x = x_0 + (\theta - x_0)\Gamma\left(1 + \frac{1}{b}\right) = 0.02 + 4.439\Gamma\left(1 + \frac{1}{1.483}\right) = 4.033$$

The median dimensionless life is, from Eq. (20–26) where $R = 0.5$,

Answer

$$x_{0.50} = x_0 + (\theta - x_0)\left(\ln\frac{1}{R}\right)^{1/b} = 0.02 + 4.439\left(\ln\frac{1}{0.5}\right)^{1/1.483}$$

$$= 3.487$$

The 10th percentile value of the dimensionless life x is

Answer

$$x_{0.10} = 0.02 + 4.439\left(\ln\frac{1}{0.90}\right)^{1/1.483} \doteq 1 \qquad \text{(as it should be)}$$

The standard deviation of the dimensionless life is given by Eq. (20–29):

Answer

$$\hat{\sigma}_x = (\theta - x_0)\left[\Gamma\left(1 + \frac{2}{b}\right) - \Gamma^2\left(1 + \frac{1}{b}\right)\right]^{1/2}$$

$$= 4.439\left[\Gamma\left(1 + \frac{2}{1.483}\right) - \Gamma^2\left(1 + \frac{1}{1.483}\right)\right]^{1/2} = 2.753$$

The coefficient of variation of the dimensionless life is

Answer

$$C_x = \frac{\hat{\sigma}_x}{\mu_x} = \frac{2.753}{4.033} = 0.683$$

11–5 Relating Load, Life, and Reliability

This is the designer's problem. The desired load is not the manufacturer's test load or catalog entry. The desired speed is different from the vendor's test speed, and the reliability expectation is typically much higher than the 0.90 accompanying the catalog entry. Figure 11–5 shows the situation. The catalog information is plotted as point A, whose coordinates are (the logs of) C_{10} and $x_{10} = L_{10}/L_{10} = 1$, a point on the 0.90 reliability contour. The design point is at D, with the coordinates (the logs of) F_D and x_D, a point that is on the $R = R_D$ reliability contour. The designer must move from point D to point A via point B as follows. Along a constant reliability contour (BD), Eq. (11–2) applies:

$$F_B x_B^{1/a} = F_D x_D^{1/a}$$

from which

$$F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a} \qquad (a)$$

Along a constant load line (AB), Eq. (11–4) applies:

$$R_D = \exp\left[-\left(\frac{x_B - x_0}{\theta - x_0}\right)^b\right]$$

Solving for x_B gives

$$x_B = x_0 + (\theta - x_0)\left(\ln\frac{1}{R_D}\right)^{1/b}$$

Now substitute this in Eq. (a) to obtain

$$F_B = F_D\left(\frac{x_D}{x_B}\right)^{1/a} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}}\right]^{1/a}$$

However, $F_B = C_{10}$, so

$$C_{10} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}}\right]^{1/a} \qquad (11\text{–}6)$$

As useful as Eq. (11–6) is, one's attention to keystrokes and their sequence on a handheld calculator strays, and, as a result, the most common error is keying in the inappropriate logarithm. We have the opportunity here to make Eq. (11–6) more error-proof. Note that

$$\ln \frac{1}{R_D} = \ln \frac{1}{1 - p_f} = \ln(1 + p_f + \cdots) \doteq p_f = 1 - R_D$$

where p_f is the probability for failure. Equation (11–6) can be written as

$$C_{10} \doteq F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \qquad R \geq 0.90 \qquad (11\text{–}7)$$

Loads are often nonsteady, so that the desired load is multiplied by an application factor a_f. The steady load $a_f F_D$ does the same damage as the variable load F_D does to the rolling surfaces. This point will be elaborated later.

EXAMPLE 11–3

The design load on a ball bearing is 413 lbf and an application factor of 1.2 is appropriate. The speed of the shaft is to be 300 rev/min, the life to be 30 kh with a reliability of 0.99. What is the C_{10} catalog entry to be sought (or exceeded) when searching for a deep-groove bearing in a manufacturer's catalog on the basis of 10^6 revolutions for rating life? The Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$.

Solution

$$x_D = \frac{L}{L_{10}} = \frac{60 L_D n_D}{60 L_R n_R} = \frac{60(30\ 000)300}{10^6} = 540$$

Thus, the design life is 540 times the L_{10} life. For a ball bearing, $a = 3$. Then, from Eq. (11–7),

Answer

$$C_{10} = (1.2)(413) \left[\frac{540}{0.02 + 4.439(1 - 0.99)^{1/1.483}} \right]^{1/3} = 6696 \text{ lbf}$$

We have learned to identify the catalog basic load rating corresponding to a steady radial load F_D, a desired life L_D, and a speed n_D.

Shafts generally have two bearings. Often these bearings are different. If the bearing reliability of the shaft with its pair of bearings is to be R, then R is related to the individual bearing reliabilities R_A and R_B by

$$R = R_A R_B$$

First, we observe that if the product $R_A R_B$ equals R, then, in general, R_A and R_B are both greater than R. Since the failure of either or both of the bearings results in the shutdown of the shaft, then A or B or both can create a failure. Second, in sizing bearings one can begin by making R_A and R_B equal to the square root of the reliability goal, $\sqrt{R}$. In Ex. 11–3, if the bearing was one of a pair, the reliability goal would be $\sqrt{0.99}$, or 0.995. The bearings selected are discrete in their reliability property in your problem, so the selection procedure "rounds up," and the overall reliability exceeds the goal R. Third, it may be possible, if $R_A > \sqrt{R}$, to round down on B yet have the product $R_A R_B$ still exceed the goal R.

11–6 Combined Radial and Thrust Loading

A ball bearing is capable of resisting radial loading and a thrust loading. Furthermore, these can be combined. Consider F_a and F_r to be the axial thrust and radial loads, respectively, and F_e to be the *equivalent radial load* that does the same damage as the combined radial and thrust loads together. A rotation factor V is defined such that $V = 1$ when the inner ring rotates and $V = 1.2$ when the outer ring rotates. Two dimensionless groups can now be formed: F_e/VF_r and F_a/VF_r. When these two dimensionless groups are plotted as in Fig. 11–6, the data fall in a gentle curve that is well approximated by two straight-line segments. The abscissa e is defined by the intersection of the two lines. The equations for the two lines shown in Fig. 11–6 are

$$\frac{F_e}{VF_r} = 1 \qquad \text{when } \frac{F_a}{VF_r} \le e \tag{11–8a}$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \qquad \text{when } \frac{F_a}{VF_r} > e \tag{11–8b}$$

where, as shown, X is the ordinate intercept and Y is the slope of the line for $F_a/VF_r > e$. It is common to express Eqs. (11–8*a*) and (11–8*b*) as a single equation,

$$F_e = X_i V F_r + Y_i F_a \tag{11–9}$$

where $i = 1$ when $F_a/VF_r \le e$ and $i = 2$ when $F_a/VF_r > e$. Table 11–1 lists values of X_1, Y_1, X_2, and Y_2 as a function of e, which in turn is a function of F_a/C_0, where C_0 is the bearing static load catalog rating.

In these equations, the rotation factor V is intended to correct for the rotating-ring conditions. The factor of 1.2 for outer-ring rotation is simply an acknowledgment that the fatigue life is reduced under these conditions. Self-aligning bearings are an exception: they have $V = 1$ for rotation of either ring.

The X and Y factors in Eqs. (11–8*a*) and (11–8*b*) depend upon the geometry of the bearing, including the number of balls and the ball diameter. The ABMA

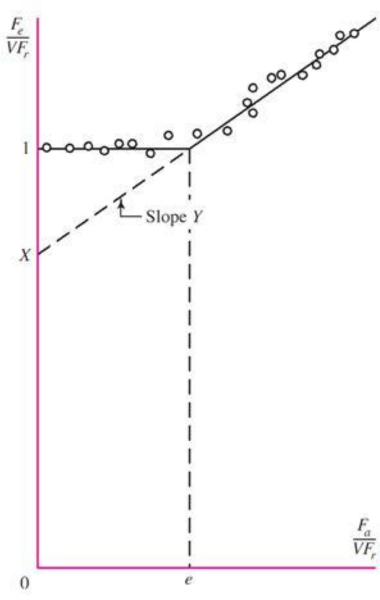


Figure 11–6

The relationship of dimensionless group $F_e/(VF_r)$ and $F_a/(VF_r)$ and the straight-line segments representing the data.

Table 11–1

Equivalent Radial Load Factors for Ball Bearings

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

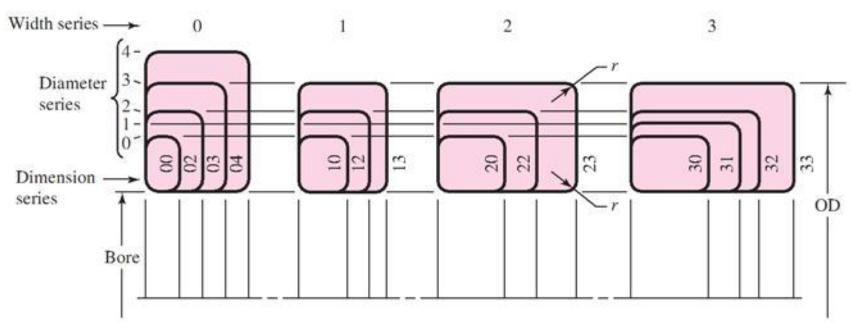


Figure 11–7

The basic ABMA plan for boundary dimensions. These apply to ball bearings, straight roller bearings, and spherical roller bearings, but not to inch-series ball bearings or tapered roller bearings. The contour of the corner is not specified. It may be rounded or chamfered, but it must be small enough to clear the fillet radius specified in the standards.

recommendations are based on the ratio of the thrust component F_a to the *basic static load rating* C_0 and a variable reference value e. The static load rating C_0 is tabulated, along with the basic dynamic load rating C_{10}, in many of the bearing manufacturers' publications; see Table 11–2, for example.

Since straight or cylindrical roller bearings will take no axial load, or very little, the Y factor is always zero.

The ABMA has established standard boundary dimensions for bearings, which define the bearing bore, the outside diameter (OD), the width, and the fillet sizes on the shaft and housing shoulders. The basic plan covers all ball and straight roller bearings in the metric sizes. The plan is quite flexible in that, for a given bore, there is an assortment of widths and outside diameters. Furthermore, the outside diameters selected are such that, for a particular outside diameter, one can usually find a variety of bearings having different bores and widths.

This basic ABMA plan is illustrated in Fig. 11–7. The bearings are identified by a two-digit number called the *dimension-series code*. The first number in the code is from the *width series*, 0, 1, 2, 3, 4, 5, and 6. The second number is from the *diameter series*

Table 11–2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_S	d_H	Load Ratings, kN Deep Groove C_{10}	C_0	Angular Contact C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

(outside), 8, 9, 0, 1, 2, 3, and 4. Figure 11–7 shows the variety of bearings that may be obtained with a particular bore. Since the dimension-series code does not reveal the dimensions directly, it is necessary to resort to tabulations. The 02 series is used here as an example of what is available. See Table 11–2.

The housing and shaft shoulder diameters listed in the tables should be used whenever possible to secure adequate support for the bearing and to resist the maximum thrust loads (Fig. 11–8). Table 11–3 lists the dimensions and load ratings of some straight roller bearings.

To assist the designer in the selection of bearings, most of the manufacturers' handbooks contain data on bearing life for many classes of machinery, as well as information on load-application factors. Such information has been accumulated the hard way, that is, by experience, and the beginner designer should utilize this information until he or she gains enough experience to know when deviations are possible. Table 11–4 contains recommendations on bearing life for some classes of machinery. The load-application factors in Table 11–5 serve the same purpose as factors of safety; use them to increase the equivalent load before selecting a bearing.

Figure 11–8

Shaft and housing shoulder diameters d_S and d_H should be adequate to ensure good bearing support.

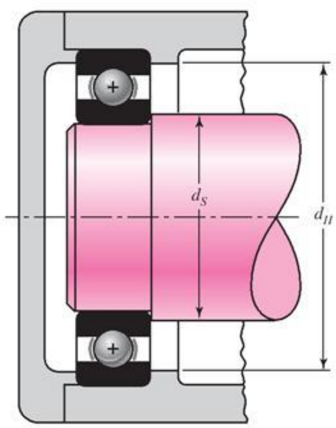


Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Table 11–4

Bearing-Life Recommendations for Various Classes of Machinery

Type of Application	Life, kh
Instruments and apparatus for infrequent use	Up to 0.5
Aircraft engines	0.5–2
Machines for short or intermittent operation where service interruption is of minor importance	4–8
Machines for intermittent service where reliable operation is of great importance	8–14
Machines for 8-h service that are not always fully utilized	14–20
Machines for 8-h service that are fully utilized	20–30
Machines for continuous 24-h service	50–60
Machines for continuous 24-h service where reliability is of extreme importance	100–200

Table 11–5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

The static load rating is given in bearing catalog tables. It comes from the equations

$$C_0 = M n_b d_b^2 \qquad \text{(ball bearings)}$$

and

$$C_0 = M n_r l_c d \qquad \text{(roller bearings)}$$

where C_0 = bearing static load rating, lbf (kN)
n_b = number of balls
n_r = number of rollers
d_b = diameter of balls, in (mm)
d = diameter of rollers, in (mm)
l_c = length of contact line, in (mm)

and M takes on the values of which the following table is representative:

M	in and lbf	mm and kN
Radial ball	$1.78(10)^3$	$5.11(10)^3$
Ball thrust	$7.10(10)^3$	$20.4(10)^3$
Radial roller	$3.13(10)^3$	$8.99(10)^3$
Roller thrust	$14.2(10)^3$	$40.7(10)^3$

EXAMPLE 11–4 An SKF 6210 angular-contact ball bearing has an axial load F_a of 400 lbf and a radial load F_r of 500 lbf applied with the outer ring stationary. The basic static load rating C_0 is 4450 lbf and the basic load rating C_{10} is 7900 lbf. Estimate the L_{10} life at a speed of 720 rev/min.

Solution $V = 1$ and $F_a/C_0 = 400/4450 = 0.090$. Interpolate for e in Table 11–1:

F_a/C_0	e	
0.084	0.28	
0.090	e	from which $e = 0.285$
0.110	0.30	

$F_a/(VF_r) = 400/[(1)500] = 0.8 > 0.285$. Thus, interpolate for Y_2:

F_a/C_0	Y_2	
0.084	1.55	
0.090	Y_2	from which $Y_2 = 1.527$
0.110	1.45	

From Eq. (11–9),

$$F_e = X_2VF_r + Y_2F_a = 0.56(1)500 + 1.527(400) = 890.8\ \text{lbf}$$

With $L_D = L_{10}$ and $F_D = F_e$, solving Eq. (11–3) for L_{10} gives

Answer

$$L_{10} = \frac{60L_Rn_R}{60n_D}\left(\frac{C_{10}}{F_e}\right)^a = \frac{10^6}{60(720)}\left(\frac{7900}{890.8}\right)^3 = 16\,150\ \text{h}$$

We now know how to combine a steady radial load and a steady thrust load into an equivalent steady radial load F_e that inflicts the same damage per revolution as the radial–thrust combination.

11–7 Variable Loading

Bearing loads are frequently variable and occur in some identifiable patterns:

- Piecewise constant loading in a cyclic pattern
- Continuously variable loading in a repeatable cyclic pattern
- Random variation

Equation (11–1) can be written as

$$F^aL = \text{constant} = K \tag{a}$$

Note that F may already be an equivalent steady radial load for a radial–thrust load combination. Figure 11–9 is a plot of F^a as ordinate and L as abscissa for Eq. (a). If a load level of F_1 is selected and run to the failure criterion, then the area under the F_1-L_1 trace is numerically equal to K. The same is true for a load level F_2; that is, the area under the F_2-L_2 trace is numerically equal to K. The linear damage theory says that in the case of

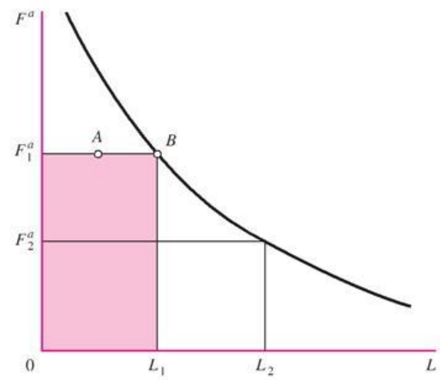


Figure 11–9

Plot of F^a as ordinate and L as abscissa for F^aL = constant. The linear damage hypothesis says that in the case of load F_1, the area under the curve from $L = 0$ to $L = L_A$ is a measure of the damage $D = F_1^a L_A$. The complete damage to failure is measured by $C_{10}^a L_B$.

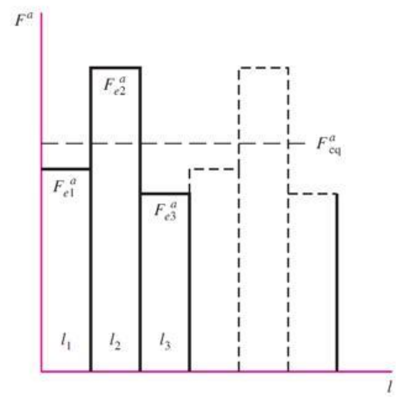


Figure 11–10

A three-part piecewise-continuous periodic loading cycle involving loads F_{e1}, F_{e2}, and F_{e3}. F_{eq} is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage D per period.

Consider the piecewise continuous cycle depicted in Fig. 11–10. The loads F_{ei} are equivalent steady radial loads for combined radial–thrust loads. The damage done by loads F_{e1}, F_{e2}, and F_{e3} is

$$D = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 \tag{b}$$

where l_i is the number of revolutions at life L_i. The equivalent steady load F_{eq} when run for $l_1 + l_2 + l_3$ revolutions does the same damage D. Thus

$$D = F_{eq}^a (l_1 + l_2 + l_3) \tag{c}$$

Equating Eqs. (*b*) and (*c*), and solving for F_{eq}, we get

$$F_{\text{eq}} = \left[\frac{F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3}{l_1 + l_2 + l_3}\right]^{1/a} = \left[\sum f_i F_{ei}^a\right]^{1/a} \tag{11–10}$$

where f_i is the fraction of revolution run up under load F_{ei}. Since l_i can be expressed as $n_i t_i$, where n_i is the rotational speed at load F_{ei} and t_i is the duration of that speed, then it follows that

$$F_{\text{eq}} = \left[\frac{\sum n_i t_i F_{ei}^a}{\sum n_i t_i}\right]^{1/a} \tag{11–11}$$

The character of the individual loads can change, so an application factor (a_f) can be prefixed to each F_{ei} as $(a_{fi} F_{ei})^a$; then Eq. (11–10) can be written

$$F_{\text{eq}} = \left[\sum f_i (a_{fi} F_{ei})^a\right]^{1/a} \qquad L_{\text{eq}} = \frac{K}{F_{\text{eq}}^a} \tag{11–12}$$

EXAMPLE 11–5 A ball bearing is run at four piecewise continuous steady loads as shown in the following table. Columns (1), (2), and (5) to (8) are given.

(1) Time Fraction	(2) Speed, rev/min	(3) Product, Column (1) × (2)	(4) Turns Fraction, (3)/∑(3)	(5) F_{ri}, lbf	(6) F_{ai}, lbf	(7) F_{ei}, lbf	(8) a_{fi}	(9) $a_{fi} F_{ei}$, lbf
0.1	2000	200	0.077	600	300	794	1.10	873
0.1	3000	300	0.115	300	300	626	1.25	795
0.3	3000	900	0.346	750	300	878	1.10	966
0.5	2400	1200	0.462	375	300	668	1.25	835
		2600	1.000					

Columns 1 and 2 are multiplied to obtain column 3. The column 3 entry is divided by the sum of column 3, 2600, to give column 4. Columns 5, 6, and 7 are the radial, axial, and equivalent loads respectively. Column 8 is the appropriate application factor. Column 9 is the product of columns 7 and 8.

Solution From Eq. (11–10), with $a = 3$, the equivalent radial load F_e is

Answer

$$F_e = \left[0.077(873)^3 + 0.115(795)^3 + 0.346(966)^3 + 0.462(835)^3\right]^{1/3} = 884 \text{ lbf}$$

Sometimes the question after several levels of loading is: How much life is left if the next level of stress is held until failure? Failure occurs under the linear damage hypothesis when the damage D equals the constant $K = F^a L$. Taking the first form of Eq. (11–10), we write

$$F_{\text{eq}}^a L_{\text{eq}} = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3$$

and note that

$$K = F_{e1}^a L_1 = F_{e2}^a L_2 = F_{e3}^a L_3$$

and K also equals

$$K = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3 = \frac{K}{L_1} l_1 + \frac{K}{L_2} l_2 + \frac{K}{L_3} l_3 = K \sum \frac{l_i}{L_i}$$

From the outer parts of the preceding equation we obtain

$$\sum \frac{l_i}{L_i} = 1 \tag{11–13}$$

This equation was advanced by Palmgren in 1924, and again by Miner in 1945. See Eq. (6–58), p. 315.

The second kind of load variation mentioned is continuous, periodic variation, depicted by Fig. 11–11. The differential damage done by F^a during rotation through the angle $d\theta$ is

$$dD = F^a \, d\theta$$

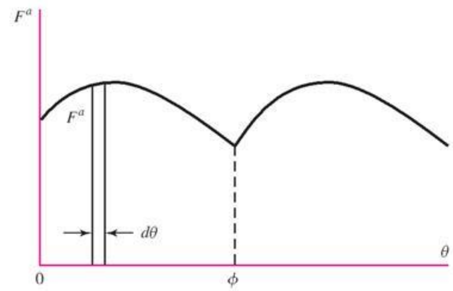


Figure 11–11

A continuous load variation of a cyclic nature whose period is ϕ.

An example of this would be a cam whose bearings rotate with the cam through the angle $d\theta$. The total damage during a complete cam rotation is given by

$$D = \int dD = \int_0^{\phi} F^a \, d\theta = F_{\text{eq}}^a \phi$$

from which, solving for the equivalent load, we obtain

$$F_{\text{eq}} = \left[\frac{1}{\phi}\int_0^{\phi} F^a \, d\theta\right]^{1/a} \qquad L_{\text{eq}} = \frac{K}{F_{\text{eq}}^a} \tag{11–14}$$

The value of ϕ is often 2π, although other values occur. Numerical integration is often useful to carry out the indicated integration, particularly when a is not an integer and trigonometric functions are involved. We have now learned how to find the steady equivalent load that does the same damage as a continuously varying cyclic load.

EXAMPLE 11–6

The operation of a particular rotary pump involves a power demand of $P = \bar{P} + A' \sin\theta$ where $\bar{P}$ is the average power. The bearings feel the same variation as $F = \bar{F} + A\sin\theta$. Develop an application factor a_f for this application of ball bearings.

Solution

From Eq. (11–14), with $a = 3$,

$$F_{\text{eq}} = \left(\frac{1}{2\pi}\int_0^{2\pi} F^a \, d\theta\right)^{1/a} = \left(\frac{1}{2\pi}\int_0^{2\pi} (\bar{F} + A\sin\theta)^3 \, d\theta\right)^{1/3}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{2\pi} \bar{F}^3 \, d\theta + 3\bar{F}^2 A \int_0^{2\pi} \sin\theta \, d\theta + 3\bar{F} A^2 \int_0^{2\pi} \sin^2\theta \, d\theta + A^3 \int_0^{2\pi} \sin^3\theta \, d\theta\right)\right]^{1/3}$$

$$F_{\text{eq}} = \left[\frac{1}{2\pi}(2\pi\bar{F}^3 + 0 + 3\pi\bar{F}A^2 + 0)\right]^{1/3} = \bar{F}\left[1 + \frac{3}{2}\left(\frac{A}{\bar{F}}\right)^2\right]^{1/3}$$

In terms of $\bar{F}$, the application factor is

Answer

$$a_f = \left[1 + \frac{3}{2}\left(\frac{A}{\bar{F}}\right)^2\right]^{1/3}$$

We can present the result in tabular form:

$A/\bar{F}$	a_f
0	1
0.2	1.02
0.4	1.07
0.6	1.15
0.8	1.25
1.0	1.36

11–8 Selection of Ball and Cylindrical Roller Bearings

We have enough information concerning the loading of rolling-contact ball and roller bearings to develop the steady equivalent radial load that will do as much damage to the bearing as the existing loading. Now let's put it to work.

EXAMPLE 11–7

The second shaft on a parallel-shaft 25-hp foundry crane speed reducer contains a helical gear with a pitch diameter of 8.08 in. Helical gears transmit components of force in the tangential, radial, and axial directions (see Chap. 13). The components of the gear force transmitted to the second shaft are shown in Fig. 11–12, at point *A*. The bearing reactions at *C* and *D*, assuming simple-supports, are also shown. A ball bearing is to be selected for location *C* to accept the thrust, and a cylindrical roller

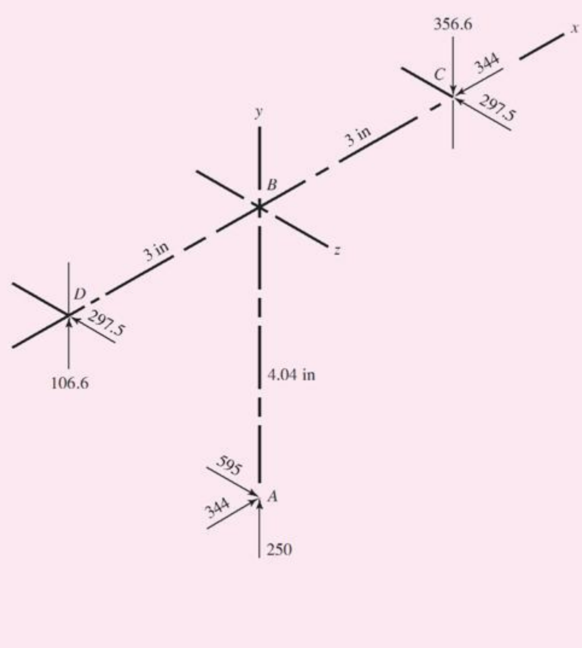


Figure 11–12

Forces in pounds applied to the second shaft of the helical gear speed reducer of Ex. 11–7.

bearing is to be utilized at location D. The life goal of the speed reducer is 10 kh, with a reliability factor for the ensemble of all four bearings (both shafts) to equal or exceed 0.96 for the Weibull parameters of Ex. 11–3. The application factor is to be 1.2.
(*a*) Select the roller bearing for location D.
(*b*) Select the ball bearing (angular contact) for location C, assuming the inner ring rotates.

Solution The torque transmitted is $T = 595(4.04) = 2404$ lbf · in. The speed at the rated horsepower, given by Eq. (3–40), p. 138, is

$$n_D = \frac{63\,025H}{T} = \frac{63\,025(25)}{2404} = 655.4 \text{ rev/min}$$

The radial load at D is $\sqrt{106.6^2 + 297.5^2} = 316.0$ lbf, and the radial load at C is $\sqrt{356.6^2 + 297.5^2} = 464.4$ lbf. The individual bearing reliabilities, if equal, must be at least $\sqrt[4]{0.96} = 0.98985 \doteq 0.99$. The dimensionless design life for both bearings is

$$x_D = \frac{L}{L_{10}} = \frac{60L_D n_D}{60L_R n_R} = \frac{60(10\,000)655.4}{10^6} = 393.2$$

(*a*) From Eq. (11–7), the Weibull parameters of Ex. 11–3, an application factor of 1.2, and $a = 10/3$ for the roller bearing at D, the catalog rating should be equal to or greater than

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$= 1.2(316.0)\left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{3/10} = 3591 \text{ lbf} = 16.0 \text{ kN}$$

Answer The absence of a thrust component makes the selection procedure simple. Choose a 02-25 mm series, or a 03-25 mm series cylindrical roller bearing from Table 11–3.

(*b*) The ball bearing at C involves a thrust component. This selection procedure requires an iterative procedure. Assuming $F_a/(VF_r) > e$,

1 Choose Y_2 from Table 11–1.
2 Find C_{10}.
3 Tentatively identify a suitable bearing from Table 11–2, note C_0.
4 Using F_a/C_0 enter Table 11–1 to obtain a new value of Y_2.
5 Find C_{10}.
6 If the same bearing is obtained, stop.
7 If not, take next bearing and go to step 4.

As a first approximation, take the middle entry from Table 11–1:

$$X_2 = 0.56 \qquad Y_2 = 1.63.$$

From Eq. (11–8*b*), with $V = 1$,

$$\frac{F_e}{VF_r} = X + \frac{Y}{V}\frac{F_a}{F_r} = 0.56 + 1.63\frac{344}{(1)464.4} = 1.77$$

$$F_e = 1.77VF_r = 1.77(1)464.4 = 822 \text{ lbf} \qquad \text{or} \qquad 3.66 \text{ kN}$$

From Eq. (11–7), with $a = 3$,

$$C_{10} = 1.2(3.66)\left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3} = 53.4 \text{ kN}$$

From Table 11–2, angular-contact bearing 02-60 mm has $C_{10} = 55.9$ kN. C_0 is 35.5 kN. Step 4 becomes, with F_a in kN,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{35.5} = 0.0431$$

which makes e from Table 11–1 approximately 0.24. Now $F_a/[VF_r] = 344/[(1)\,464.4] = 0.74$, which is greater than 0.24, so we find Y_2 by interpolation:

F_a/C_0	Y_2	
0.042	1.85	
0.043	Y_2	from which $Y_2 = 1.84$
0.056	1.71	

From Eq. (11–8*b*),

$$\frac{F_e}{VF_r} = 0.56 + 1.84\frac{344}{464.4} = 1.92$$

$$F_e = 1.92VF_r = 1.92(1)464.4 = 892 \text{ lbf} \quad \text{or} \quad 3.97 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{3.97}{3.66}53.4 = 57.9 \text{ kN}$$

From Table 11–2 an angular contact bearing 02-65 mm has $C_{10} = 63.7$ kN and C_0 of 41.5 kN. Again,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{41.5} = 0.0369$$

making e approximately 0.23. Now from before, $F_a/VF_r = 0.74$, which is greater than 0.23. We find Y_2 again by interpolation:

F_a/C_0	Y_2	
0.028	1.99	
0.0369	Y_2	from which $Y_2 = 1.90$
0.042	1.85	

From Eq. (11–8*b*),

$$\frac{F_e}{VF_r} = 0.56 + 1.90\frac{344}{464.4} = 1.967$$

$$F_e = 1.967VF_r = 1.967(1)464.4 = 913.5 \text{ lbf} \quad \text{or} \quad 4.065 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{4.07}{3.66} 53.4 = 59.4 \text{ kN}$$

Answer From Table 11–2 an angular-contact 02-65 mm is still selected, so the iteration is complete.

11–9 Selection of Tapered Roller Bearings

Tapered roller bearings have a number of features that make them complicated. As we address the differences between tapered roller and ball and cylindrical roller bearings, note that the underlying fundamentals are the same, but that there are differences in detail. Moreover, bearing and cup combinations are not necessarily priced in proportion to capacity. Any catalog displays a mix of high-production, low-production, and successful special-order designs. Bearing suppliers have computer programs that will take your problem descriptions, give intermediate design assessment information, and list a number of satisfactory cup-and-cone combinations in order of decreasing cost. Company sales offices provide access to comprehensive engineering services to help designers select and apply their bearings. At a large original equipment manufacturer's plant, there may be a resident bearing company representative.

Take a few minutes to go to your department's design library and look at a bearing supplier's engineering catalog, such as The Timken Company's *Bearing Selection Handbook—Revised* (1986). There is a log of engineering information and detail, based on long and successful experience. All we can do here is introduce the vocabulary, show congruence to fundamentals that were learned earlier, offer examples, and develop confidence. Finally, problems should reinforce the learning experience.

Form

The four components of a tapered roller bearing assembly are the

- Cone (inner ring)
- Cup (outer ring)
- Tapered rollers
- Cage (spacer-retainer)

The assembled bearing consists of two separable parts: (1) the cone assembly: the cone, the rollers, and the cage; and (2) the cup. Bearings can be made as single-row, two-row, four-row, and thrust-bearing assemblies. Additionally, auxiliary components such as spacers and closures can be used.

A tapered roller bearing can carry both radial and thrust (axial) loads, or any combination of the two. However, even when an external thrust load is not present, the radial load will induce a thrust reaction within the bearing because of the taper. To avoid the separation of the races and the rollers, this thrust must be resisted by an equal and opposite force. One way of generating this force is to always use at least two tapered roller bearings on a shaft. Two bearings can be mounted with the cone backs facing each other, in a configuration called *direct mounting,* or with the cone fronts facing each other, in what is called *indirect mounting.* Figure 11–13 shows the nomenclature of a tapered roller bearing, and the point G through which radial and axial components of load act.