Digital Control

Discrete Equivalents

Z-Transform

Stability Criteria

Steady State Error

Design of Digital Control Systems

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(Lecture 1)

Advantages and Disadvantages

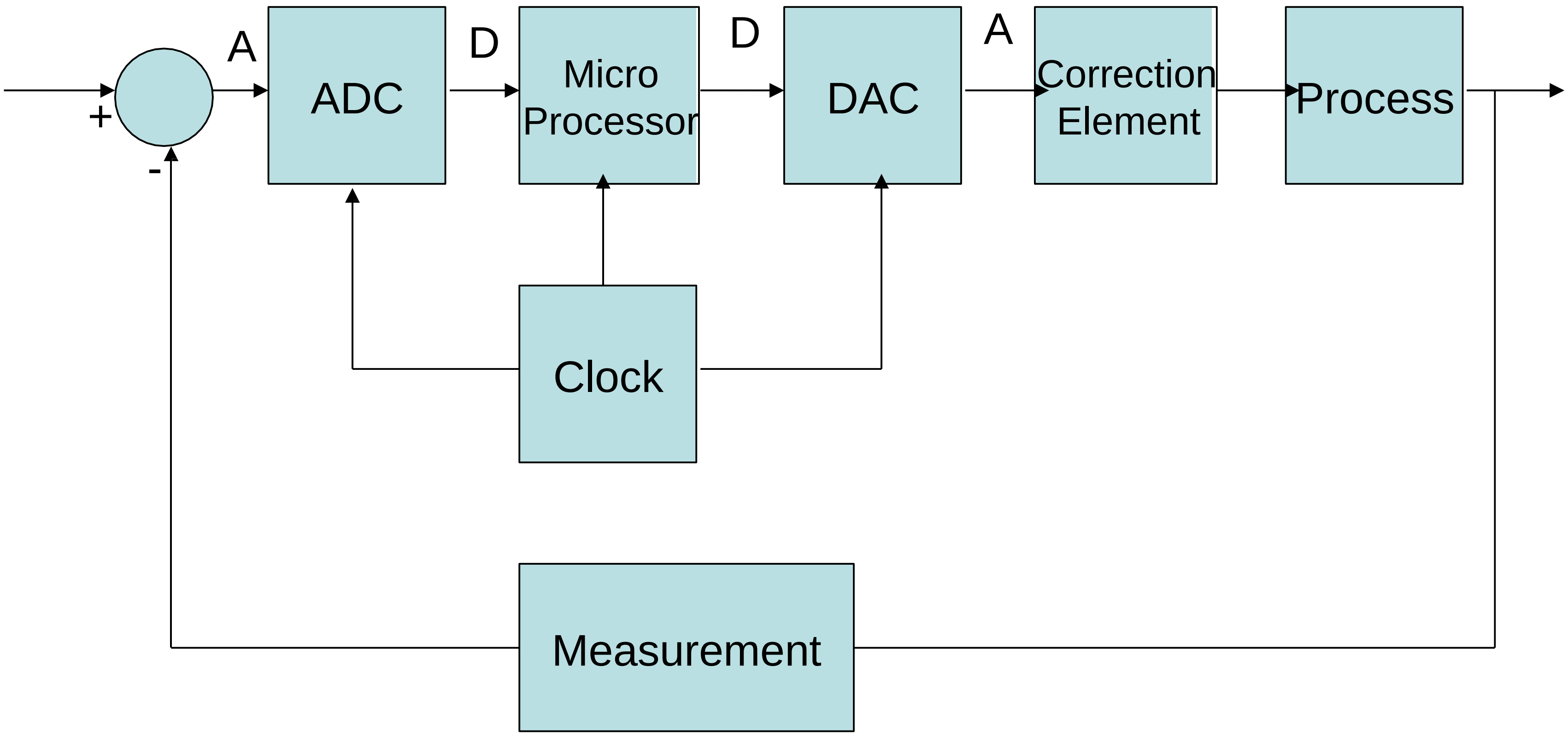
- Improved sensitivity.
- Use digital components.
- Control algorithms easily modified.
- Many systems inherently are digital.

- Develop complex math algorithms.
- Lose information during conversions due to technical problems.
- Most signals continuous in nature.

Digitization

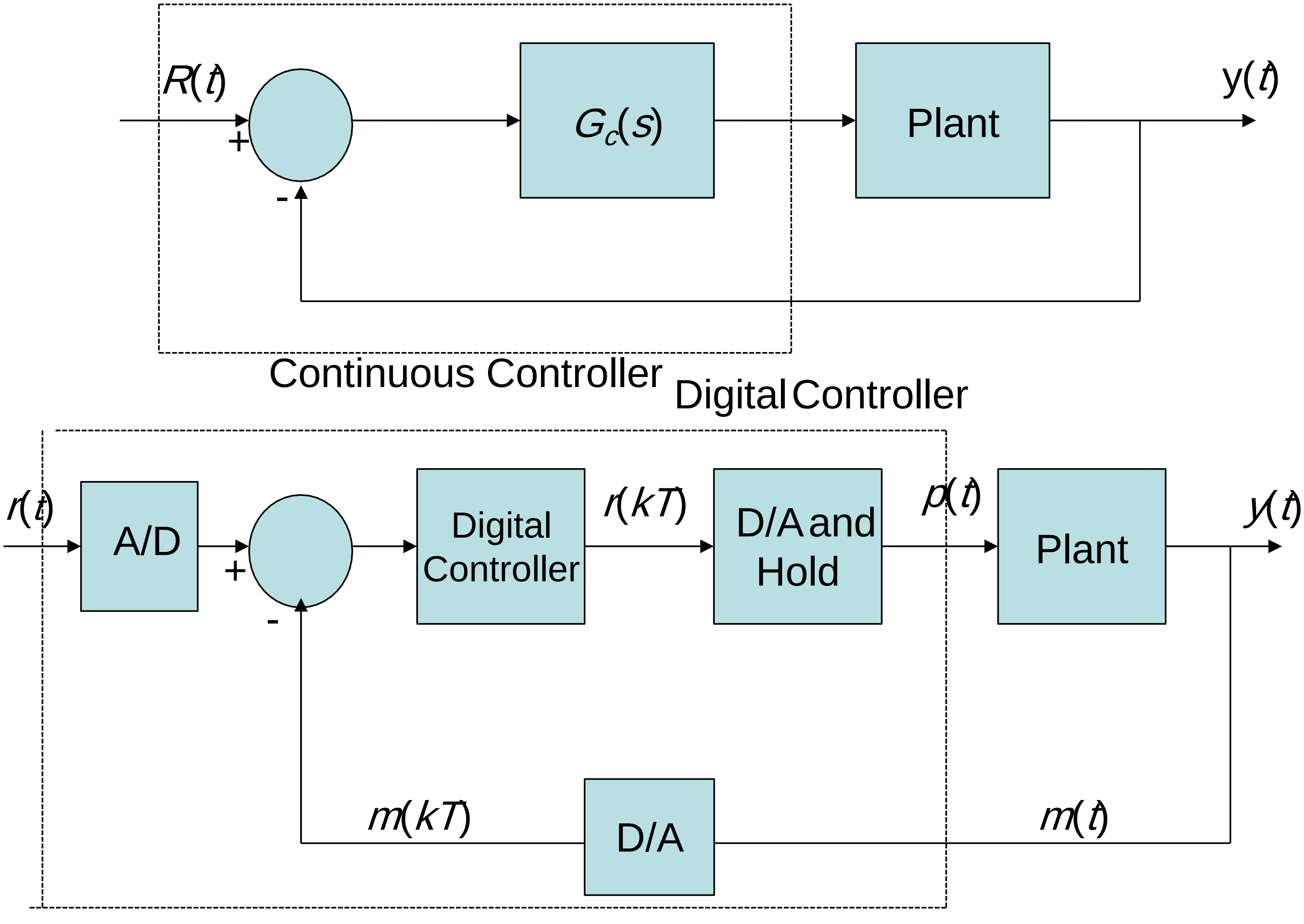
- The difference between the continuous and digital systems is that the digital system operates on samples of the sensed plant rather than the continuous signal and that the control provided by the digital controller D(s) must be generated by algebraic equations.
- In this regard, we will consider the action of the analog-to-digital (A/D) converter on the signal. This device samples a physical signal, mostly voltage, and convert it to binary number that usually consists of 10 to 16 bits.
- Conversion from the analog signal $y(t)$ to the samples $y(kt)$, occurs repeatedly at instants of time T seconds apart.
- A system having both discrete and continuous signals is called sampled data system.
- The sample rate required depends on the closed-loop bandwidth of the system. Generally, sample rates should be about 20 times the bandwidth or faster in order to assure that the digital controller will match the performance of the continuous controller.

Digital Control System



A: Analog
D: Digital

Continuous Controller and Digital Control

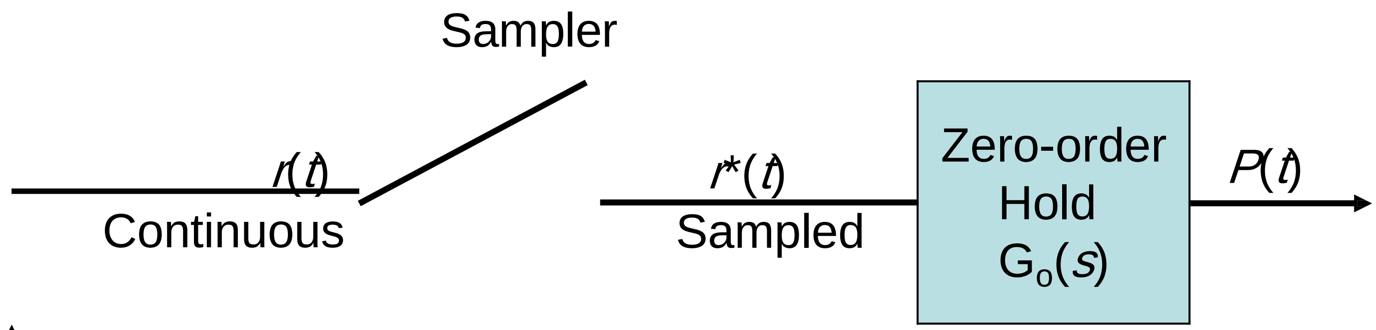


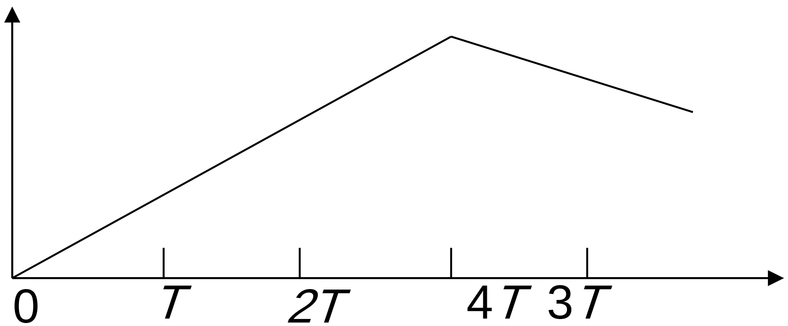
Applications of Automatic Computer Controlled Systems

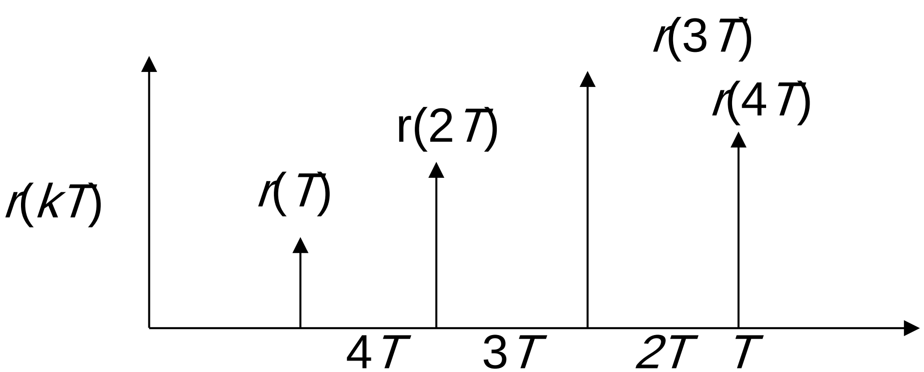
- Most control systems today use digital computers (usually microprocessors) to implement the controllers). Some applications are:
- Machine Tools
- Metal Working Processes
- Chemical Processes
- Aircraft Control
- Automobile Traffic Control
- Automobile Air-Fuel Ratio
- Digital Control Improves Sensitivity to Signal Noise.

Digital Control System

- Analog electronics can integrate and differentiate signals. In order for a digital computer to accomplish these tasks, the differential equations describing compensation must be approximated by reducing them to algebraic equations involving addition, division, and multiplication.
- A digital computer may serve as a compensator or controller in a feedback control system. Since the computer receives data only at specific intervals, it is necessary to develop a method for describing and analyzing the performance of computer control systems.
- The computer system uses data sampled at prescribed intervals, resulting in a series of signals. These time series, called sampled data, can be transformed to the *s*-domain, and then to the z-domain by the relation $z = e^{zt}$.
- Assume that all numbers that enter or leave the computer has the same fixed period *T*, called the sampling period.
- A sampler is basically a switch that closes every *T* seconds for one instant of time.

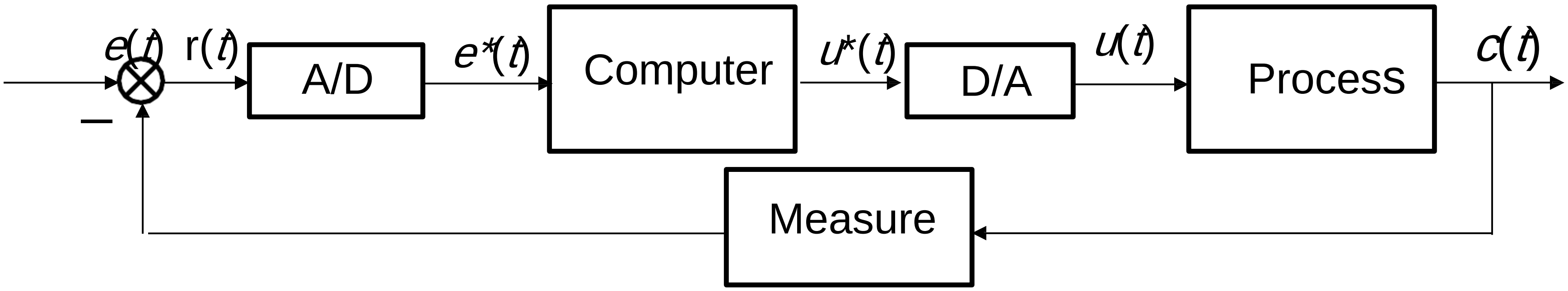






$$G_0(s) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{\left(1-e^{-sT}\right)}{s}$$

Modeling of Digital Computer



Sampling analysis

Expression of the sampling signal

$$x^*(t) = x(t)\cdot\delta_T(t) = x(t)\cdot\sum_{k=0}^{\infty}\delta(t-kT) = \sum_{k=0}^{\infty}x(kT)\delta(t-kT)$$

Analog to Digital Conversion: Sampling

An input signal is converted from continuous-varying physical value (e.g. pressure in air, or frequency or wavelength of light), by some electro-mechanical device into a continuously varying electrical signal. This signal has a range of amplitude, and a range of frequencies that can present. This continuously varying electrical signal may then be converted to a sequence of digital values, called samples, by some analog to digital conversion circuit.

- There are two factors which determine the accuracy with which the digital sequence of values captures the original continuous signal: the maximum rate at which we sample, and the number of bits used in each sample. This latter value is known as the quantization level

Zero-Order Hold

- The Zero-Order Hold block samples and holds its input for the specified sample period.
- The block accepts one input and generates one output, both of which can be scalar or vector. If the input is a vector, all elements of the vector are held for the same sample period.
- This device provides a mechanism for discretizing one or more signals in time, or resampling the signal at a different rate.
- The sample rate of the Zero-Order Hold must be set to that of the slower block. For slow-to-fast transitions, use the unit delay block.

The *z*-Transform

The z-Transform is used to take discrete time domain signals into a complex-variable frequency domain. It plays a similar role to the one the Laplace transform does in the continuous time domain. The z-transform opens up new ways of solving problems and designing discrete domain applications. The z-transform converts a discrete time domain signal, which is a sequence of real numbers, into a complex frequency domain representation.

$$r^*(t) = \sum_{k=0}^{\infty} r(kT)\,\delta(t-kT)$$

For a signal $t \rangle 0$, Using the Laplace transforms, we have

$$\Im\{r^*(t)\} = \sum_{k=0}^{\infty} r(kT)\,e^{-ksT}$$

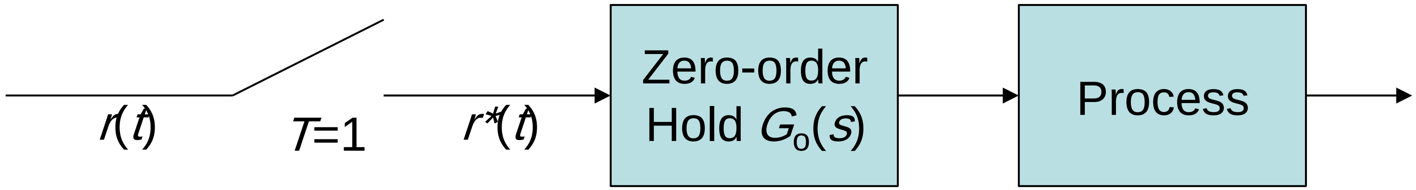
$$z = e^{sT}$$

$$Z\{r(t)\} = Z\{r^*(t)\} = \sum_{k=0}^{\infty} r(kT)\,z^{-k}$$

$$U(z) = \frac{z}{z-1}$$

$$Z\{f(t)\} = F(z) = \sum_{k=0}^{\infty} f(kT)\,z^{-k}$$

Transfer Function of Open-Loop System



$$G_o(s) = \frac{(1-e^{-st})}{s}\;;\; G_p(s) = \frac{1}{s(s+1)}$$

$$\frac{Y(s)}{R^*(s)} = G_o(s)\,G_p(s) = G(s) = \frac{1-e^{-st}}{(s+1)s^2}$$

Expanding into partial fraction : $G(s) = \left(1-e^{-st}\right)\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right)$

$$G(z) = \frac{0.3678z + 0.2644}{z^2 - 1.3678z + 0.3678}$$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{\left(1-e^{-aT}\right)z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-aT}z^{-1}\right)}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{\left(e^{-aT}-e^{-bT}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)\left(1-e^{-bT}z^{-1}\right)}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}\left(1+e^{-aT}z^{-1}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{\left[\left(aT-1+e^{-aT}\right)+\left(1-e^{-aT}-aTe^{-aT}\right)z^{-1}\right]z^{-1}}{\left(1-z^{-1}\right)^2\left(1-e^{-aT}z^{-1}\right)}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^k $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{\left(1-az^{-1}\right)^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}\left(1+az^{-1}\right)}{\left(1-az^{-1}\right)^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}\left(1+4az^{-1}+a^2z^{-2}\right)}{\left(1-az^{-1}\right)^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}\left(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3}\right)}{\left(1-az^{-1}\right)^5}$
24.	–	–	$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$