بسم الله الرحمن الرحيم

السلام عليكم ورحمة الله

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Mechanical Power Engineering Dep.

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Review of Rotational Motion and Angular Momentum

The motion of a rigid body can be considered to be the combination of the:

- **Translational motion of its center of mass and,**

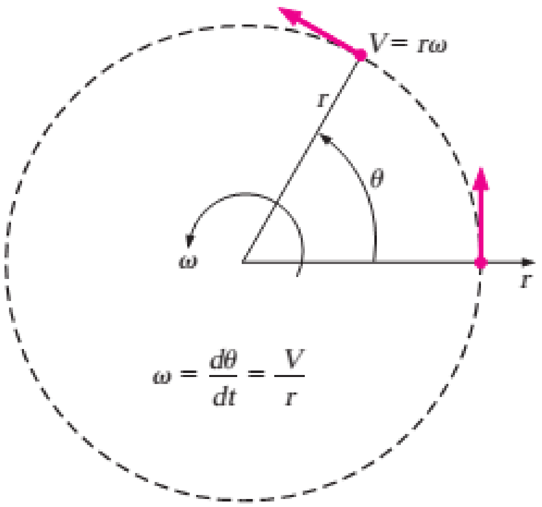
The translational motion can be analyzed using the linear momentum equation.

- **Rotational motion about its center of mass.**

all points in the body move in circles about the axis of rotation.

The amount of rotation of a point in a body is expressed in terms of the angle θ swept by a line of length r that connects the point to the axis of rotation and is perpendicular to the axis.

The physical distance traveled by a point along its circular path is $l = \theta r$, where r is the normal distance of the point from the axis of rotation and θ is the angular distance in rad. **Note that 1 rad corresponds to 360/(2π) = 57.3°.**



$$\omega = \frac{d\theta}{dt} = \frac{d(l/r)}{dt} = \frac{1}{r}\frac{dl}{dt} = \frac{V}{r}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r}\frac{dV}{dt} = \frac{a_t}{r}$$

$$V = r\omega \quad \text{and} \quad a_t = r\alpha$$

V is the linear velocity and a_t is the linear acceleration in the tangential direction for a point located at a distance r from the axis of rotation.

Newton's second law

requires that there must be a force acting in the tangential direction to cause angular acceleration.

The strength of the rotating effect, called the moment or torque, is proportional to the magnitude of the force and its distance from the axis of rotation.

$$M = rF_t = rma_t = mr^2\alpha$$

Torque: $$M = \int_{mass} r^2\alpha \, dm = \left[\int_{mass} r^2 \, dm\right]\alpha = I\alpha$$

- I is the moment of inertia of the body about the axis of rotation

Note that:

- The linear momentum of a body of mass **m** having a velocity **V** is **mV**, and the direction of linear momentum is identical to the direction of velocity.

- The moment of a force is equal to the product of the force and the normal distance, the moment of momentum, called the angular momentum.

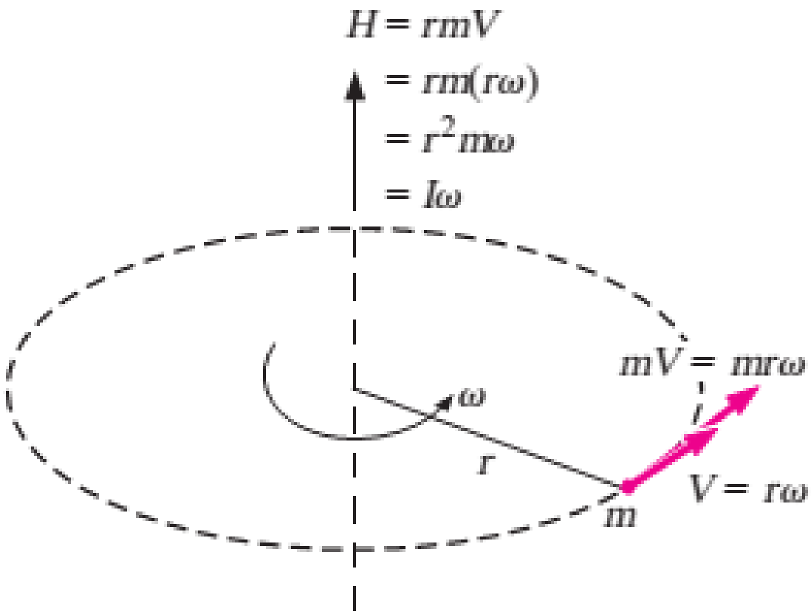
$$H = rmV = r^2 m\omega$$

Angular momentum: $H = \int_{mass} r^2\omega \, dm = \left[\int_{mass} r^2 \, dm\right]\omega = I\omega$

vector form $\vec{H} = I\vec{\omega}$

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

where $\vec{M}$ is the net torque applied on the body about the axis of rotation.

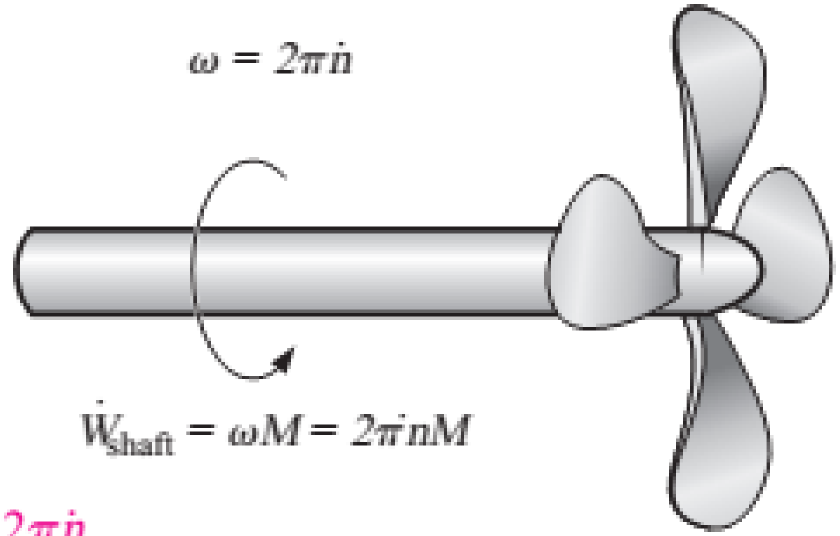


Angular momentum of point mass m rotating at angular velocity ω at distance r from the axis of rotation.

The angular velocity of rotating machinery is typically expressed in rpm and denoted by n.,

The angular velocity of rotating machinery is:

$\omega = 2\pi N/60$ rad/s or



Angular velocity versus rpm: $\omega = \frac{2\pi\dot{n}}{60}$ (rad/s)

$$\dot{W}_{shaft} = FV = Fr\omega = M\omega$$

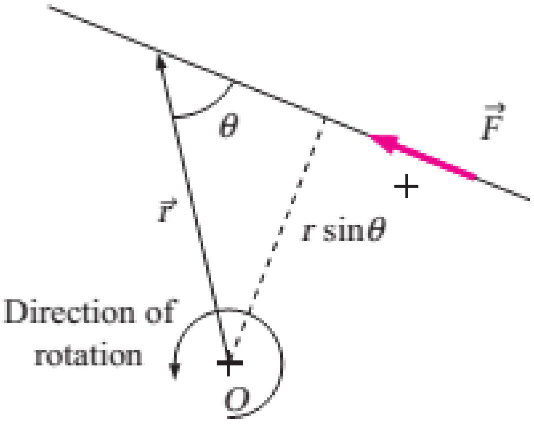
Shaft power: $\dot{W}_{shaft} = \omega M = 2\pi\dot{n}M$

The Angular Momentum Equation

angular momentum equation, also called the moment of momentum equation.

An important class of fluid devices, called turbomachines, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.

The moment of a force about a point O is the vector (or cross) product



Moment of a force: $$\vec{M} = \vec{r} \times \vec{F}$$

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$$M = Fr \sin\theta$$

Moment of momentum, also called the Angular Momentum

Moment of momentum: $\vec{H} = \vec{r} \times m\vec{V}$

$dm = \rho\, dV$ is $d\vec{H} = (\vec{r} \times \vec{V})\rho\, dV$.

Moment of momentum (system): $\vec{H}_{sys} = \int_{sys} (\vec{r} \times \vec{V})\rho\, dV$

Rate of change of moment of momentum: $\frac{d\vec{H}_{sys}}{dt} = \frac{d}{dt}\int_{sys} (\vec{r} \times \vec{V})\rho\, dV$

The angular momentum equation for a system was expressed in Eq.

$$\sum \vec{M} = \frac{d\vec{H}_{sys}}{dt} \qquad \text{where } \sum \vec{M} = \sum (\vec{r} \times \vec{F})$$

The general control volume

$$\frac{dH_{sys}}{dt} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n}) \, dA$$

General: $$\sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n}) \, dA$$

(The sum of all external moments acting on a CV) = (The time rate of change of the angular momentum of the contents of the CV) + (The net flow rate of angular momentum out of the control surface by mass flow)

$$\vec{V}_r = \vec{V} - \vec{V}_{CS}$$

For a fixed control volume (no motion or deformation of control volume) $\vec{V}_r = \vec{V}$

$$\text{Fixed CV:} \quad \sum \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V})\rho \, dV + \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n}) \, dA$$

Note that: **the forces acting on the control volume consist of body forces that act throughout the entire body of the control volume such as gravity, and surface forces that act on the control surface such as the pressure and reaction forces at points of contact.**

Special Cases

During steady flow

During steady flow, the amount of angular momentum within the control volume remains constant

Steady flow:

$$\sum \vec{M} = \int_{CS} (\vec{r} \times \vec{V})\rho(\vec{V}_r \cdot \vec{n})\, dA$$

Steady flow:

$$\sum \vec{M} = \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

In many problems, all the significant forces and momentum flows are in the same plane, and thus all give rise to moments in the same plane and about the same axis.

$$\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$$

Flow with No External Moments

No external moments:

$$0 = \frac{d\vec{H}_{CV}}{dt} + \sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}$$

This is an expression of the conservation of angular momentum principle

The rate of change of the angular momentum of a control volume is equal to the difference between the incoming and outgoing angular momentum fluxes

Radial-Flow Devices

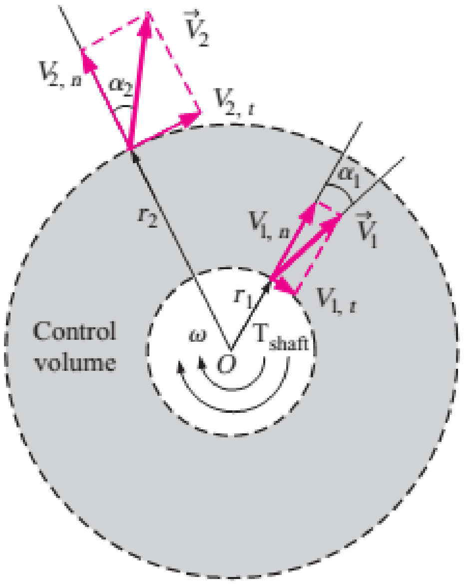
Many rotary-flow devices such as centrifugal pumps and fans involve flow in the radial direction normal to the axis of rotation and are called radial-flow devices.

To analyze the centrifugal pump

$$\dot{V}_1 = \dot{V}_2 = \dot{V} \quad \rightarrow \quad (2\pi r_1 b_1)V_{1,n} = (2\pi r_2 b_2)V_{2,n}$$

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} \quad \text{and} \quad V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2}$$

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V$$



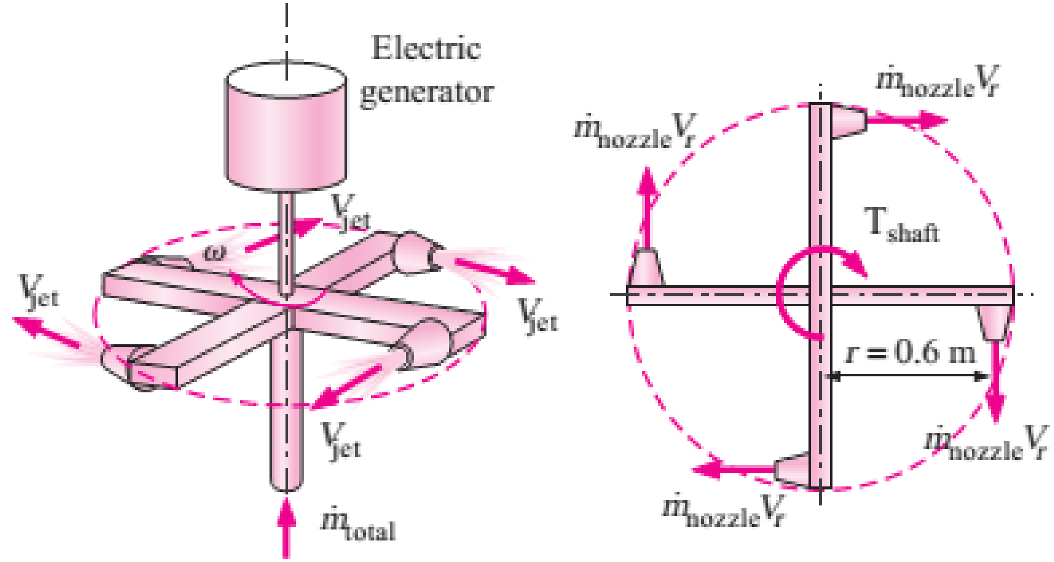
$$T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) \qquad T_{shaft} = \dot{m}(r_2 V_2 \sin\alpha_2 - r_1 V_1 \sin\alpha_1)$$

Power Generation from a Sprinkler System

Example

A large lawn sprinkler with four identical arms is to be converted into a turbine to generate electric power by attaching a generator to its rotating head, as shown in Fig. Water enters the sprinkler from the base along the axis of rotation at a rate of 20 L/s and leaves the nozzles in the tangential direction. The sprinkler rotates at a rate of 300 rpm in a horizontal plane. The diameter of each jet is 1 cm, and the normal distance between the axis of rotation and the center of each nozzle is 0.6 m. Estimate the electric power produced.

Solution



The conservation of mass equation for this steady-flow system is $m_1 = m_2 = m_{tot.}$

$\dot{m}_{nozzle} = \dot{m}_{tot.}/4$ or

$$\dot{V}_{nozzle} = \dot{V}_{total}/4$$

The average jet exit velocity relative to the nozzle is:

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{5\ \text{L/s}}{[\pi(0.01\ \text{m})^2/4]}\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right) = 63.66\ \text{m/s}$$

The angular and tangential velocities of the nozzles are

$$\omega = 2\pi\dot{n} = 2\pi(300 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 31.42 \text{ rad/s}$$

$$V_{\text{nozzle}} = r\omega = (0.6 \text{ m})(31.42 \text{ rad/s}) = 18.85 \text{ m/s}$$

The average velocity of the water jet relative to the control volume

$$V_r = V_{\text{jet}} - V_{\text{nozzle}} = 63.66 - 18.85 = 44.81 \text{ m/s}$$

The angular momentum equation

$$\sum M = \sum_{\text{out}} r\dot{m}V - \sum_{\text{in}} r\dot{m}V,$$

All moments in the counterclockwise direction are positive, and all moments in the clockwise direction are negative.

$$-\mathrm{T}_{\mathrm{shaft}} = -4r\dot{m}_{\mathrm{nozzle}}V_r \qquad \text{or} \qquad \mathrm{T}_{\mathrm{shaft}} = r\dot{m}_{\mathrm{total}}V_r$$

$$\mathrm{T}_{\mathrm{shaft}} = r\dot{m}_{\mathrm{total}}V_r = (0.6\ \mathrm{m})(20\ \mathrm{kg/s})(44.81\ \mathrm{m/s})\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg}\cdot\mathrm{m/s^2}}\right) = 537.7\ \mathrm{N}\cdot\mathrm{m}$$

$$\dot{W} = 2\pi\dot{n}\mathrm{T}_{\mathrm{shaft}} = \omega\mathrm{T}_{\mathrm{shaft}} = (31.42\ \mathrm{rad/s})(537.7\ \mathrm{N}\cdot\mathrm{m})\left(\frac{1\ \mathrm{kW}}{1000\ \mathrm{N}\cdot\mathrm{m/s}}\right) = \mathbf{16.9\ kW}$$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Thank You
For Your Attention