بسم الله الرحمن الرحيم

السلام عليكم ورحمة الله

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Faculty of Engineering
Mechanical Power Engineering Dep.

Fluid Mechanics 2

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Lecture No. 3

Momentum Analysis of Flow Systems

Objectives of this Chapter

- **Define the various kinds of forces and moments acting on a control volume**
- **Use control volume analysis to determine the forces associated with fluid flow**
- **Use control volume analysis to determine the moments caused by fluid flow and the torque transmitted**

Momentum Alysis of Flow Systems

Most engineering problems, including those associated with fluid flow, can be analyzed using one of three basic approaches:

- **Differential Approaches.**
- **Experimental Approaches.**
- **Finite Control Volume Approach**

Applications of the Momentum Equation

- **Force due to the flow of fluid a round a pipe bend.**
- **Force on a nozzle at the outlet of a pipe.**
- **Impact of a jet on a plane surface.**
- **Force due to to flow round a curved vane.**

Newton’s Laws and Conservation of Momentum

Newton’s first law states that: a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.

Newton’s second law states that: the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.

Newton’s third law states that: when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

For a rigid body of mass m, Newton's second law is expressed as:

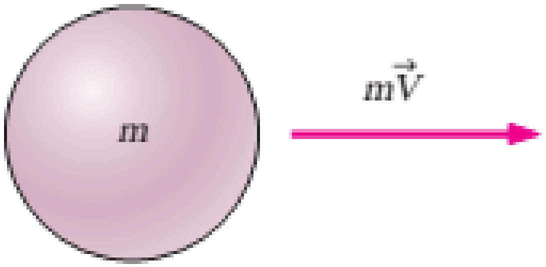
$$\text{Newton's second law:} \qquad \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt}$$

Where: $\vec{F}$ **is the net force acting on the body and** $\vec{a}$ **is the acceleration of the body under the influence of** $\vec{F}$**.**

The product of the mass and the velocity of a body is called the linear momentum or just the momentum of the body.

The momentum of a rigid body of mass m moving with a velocity $\vec{V}$ is:

Momentum = m* $\vec{V}$



The rate of change of the momentum of a body is equal to:

$$F_x = ma_x = d(mv_x)/dt$$

Newton's second law for rotating rigid bodies is:

$$\vec{M} = I\vec{\alpha}$$

$\vec{M}$ is the net moment or torque applied on the body, **I** is the moment of inertia of the body about the axis of rotation, and $\vec{\alpha}$ is the angular acceleration.

The rate of change of angular momentum:

Angular momentum equation:

$$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

where $\vec{\omega}$ is the angular velocity.

For a rigid body rotating about a fixed x-axis, the angular momentum equation can be written in scalar form as:

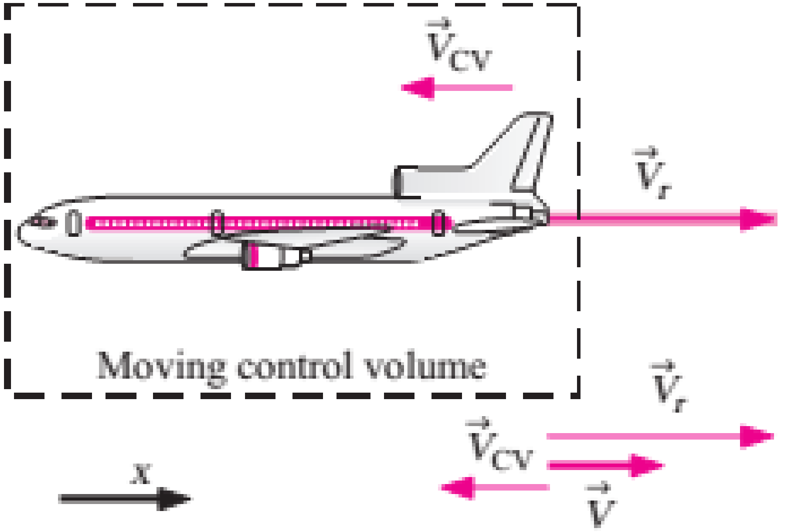
Angular momentum about x-axis:

$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

Types of Control Volume

Moving Control Volume:

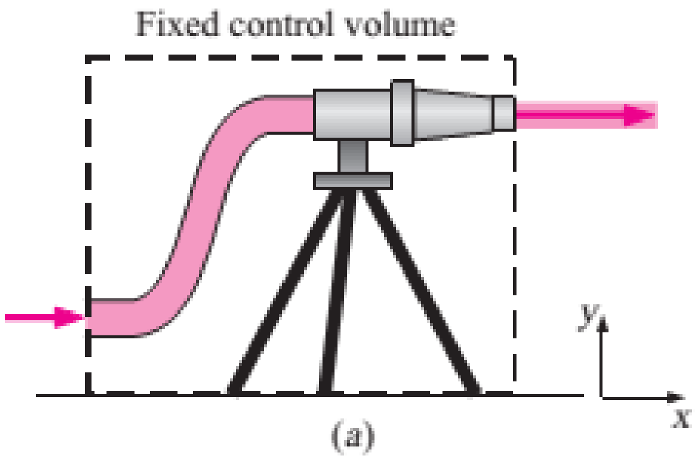
$$\vec{V}_r = \vec{V} - \vec{V}_{CV}$$



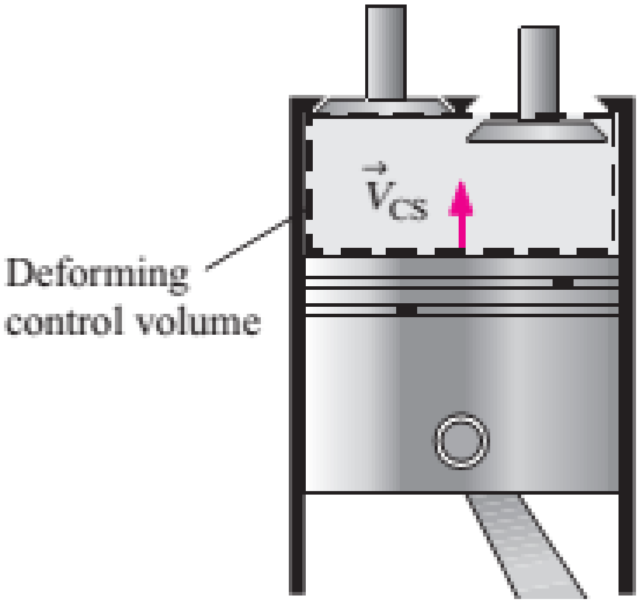
Example, if the airplane is moving at 500 km/h to the left, and the velocity of the exhaust gases is 800 km/h to the right relative to the ground, the velocity of the exhaust gases relative to the nozzle exit is:

$$\vec{V}_r = \vec{V} - \vec{V}_{CV} = 800\vec{i} - (-500\vec{i}) = 1300\vec{i} \text{ km/h}$$

Fixed Control Volume:



Deforming Control Volume:



Forces Acting on A control Volume

Body Forces:

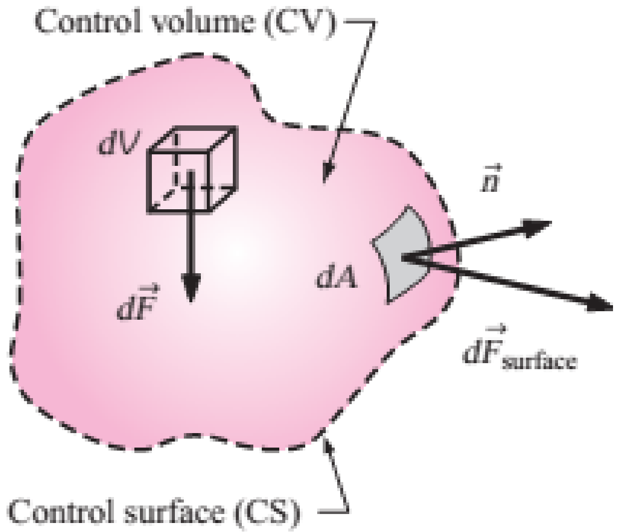
that act throughout the entire body of the control volume (such as , fluid and metal weight, gravity, electric, and magnetic forces)

Surface Forces:

that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact).

Total force acting on control volume: $$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$$

The total force acting on a control volume is composed of body forces and surface forces; body force is shown on a differential volume element, and surface force is shown on a differential surface element.



The Body Force

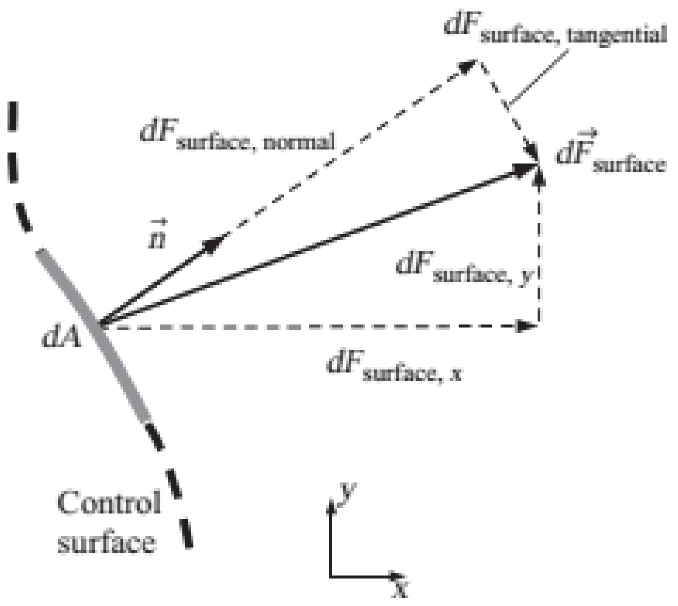
The differential body force $d\vec{F}_{body} = d\vec{F}_{gravity}$

Gravitational force acting on a fluid element: $d\vec{F}_{gravity} = \rho\vec{g}\,dV$

Gravitational vector in Cartesian coordinates: $\vec{g} = -g\vec{k}$

Total body force acting on control volume: $\sum \vec{F}_{body} = \int_{CV} \rho\vec{g}\,dV = m_{CV}\vec{g}$

The Surface Force



Surface force acting on a differential surface element: $d\vec{F}_{\text{surface}} = \sigma_{ij} \cdot \vec{n}\, dA$

Total surface force acting on control surface: $\sum \vec{F}_{\text{surface}} = \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$

$$\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}} = \int_{CV} \rho \vec{g}\, dV + \int_{CS} \sigma_{ij} \cdot \vec{n}\, dA$$

Total force: $\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$

The Linear Momentum Equation

Newton's second law for a system of mass m subjected to a net force $\vec{F}$ is

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

where $m\vec{V}$ is the **linear momentum** of the system.

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{sys}} \rho \vec{V}\, dV$$

where $\delta m = \rho dV$

Newton's second law can be stated as: the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.

General: $$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \int_{CS} \rho \vec{V}(\vec{V}_r \cdot \vec{n})\, dA$$

The product $\rho(\vec{V}_r \cdot \vec{n})\, dA$ represents the mass flow rate

$$\begin{pmatrix}\text{The sum of all}\\ \text{external forces}\\ \text{acting on a CV}\end{pmatrix} = \begin{pmatrix}\text{The time rate of change}\\ \text{of the linear momentum}\\ \text{of the contents of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{linear momentum out of the}\\ \text{control surface by mass flow}\end{pmatrix}$$

For Steady Flow

During steady flow, the amount of momentum within the control volume remains constant

$$\sum \vec{F} = \int_{CS} \rho \vec{V} (\vec{V}_r \cdot \vec{n})\, dA$$

Mass flow rate across an inlet or outlet: $\dot{m} = \int_{A_c} \rho(\vec{V} \cdot \vec{n})\, dA_c = \rho V_{avg} A_c$

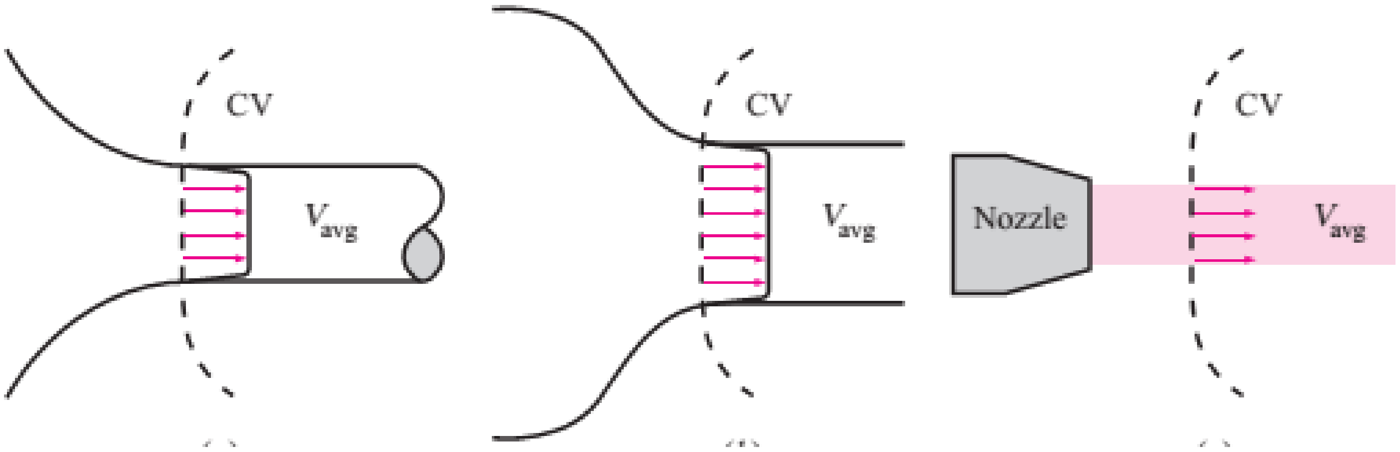
Momentum flow rate across a uniform inlet or outlet:

$$\int_{A_c} \rho \vec{V}(\vec{V} \cdot \vec{n})\, dA_c = \rho V_{avg} A_c \vec{V}_{avg} = \dot{m} \vec{V}_{avg}$$

Momentum-Flux Correction Factor, β

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V}\, dV + \sum_{out} \beta \dot{m} \vec{V}_{avg} - \sum_{in} \beta \dot{m} \vec{V}_{avg}$$

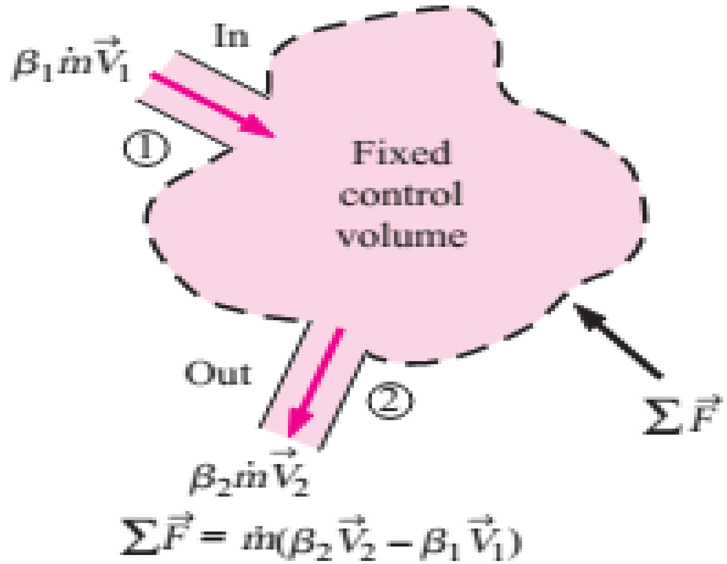
For uniform Flow; β =1.0



Momentum-flux correction factor: $$\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$$

Steady linear momentum equation: $$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

One inlet and one outlet: $$\sum \vec{F} = \dot{m} (\beta_2 \vec{V}_2 - \beta_1 \vec{V}_1)$$



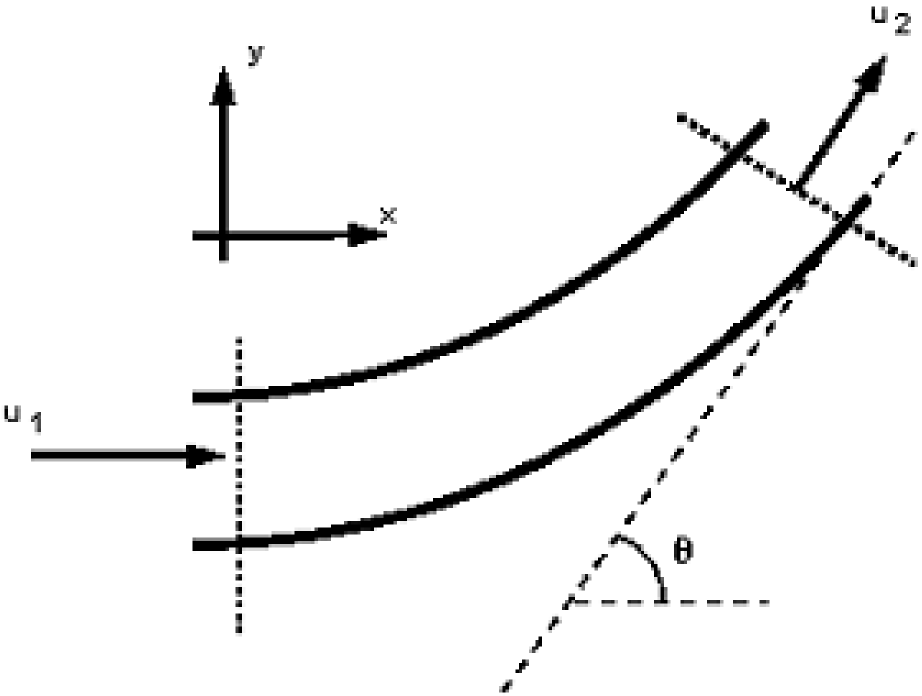
Along x-coordinate: $$\sum F_x = \dot{m}(\beta_2 V_{2,x} - \beta_1 V_{1,x})$$

Steps in Analysis

- Draw a control volume.
- Chose the co-ordinate axis system.
- Calculate the total force.
- Calculate the pressure force.
- Calculate the body force.
- Calculate the resultant force.

Example 1

Consider a pipe bend with a constant cross section lying in the horizontal plane and turning through an angle of θ .

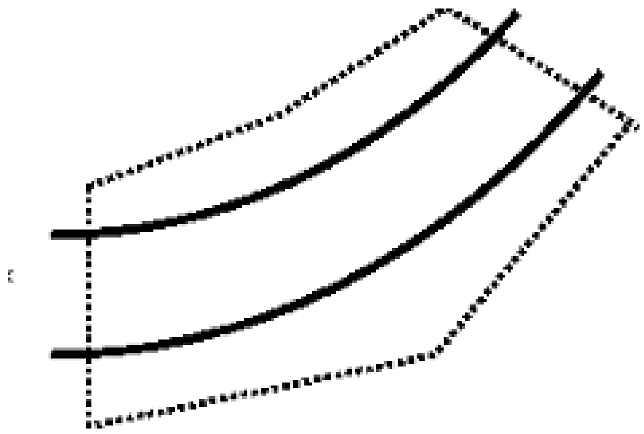


Flow round a pipe bend of constant cross-section

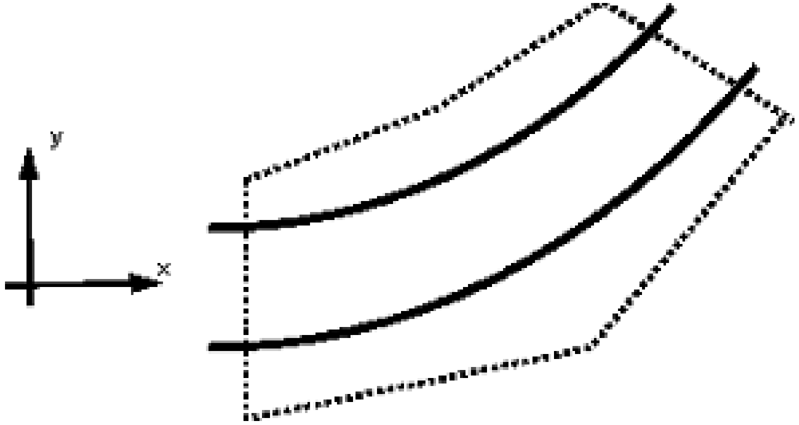
Solution

Step in Analysis:

1. Control Volume:



2. Co-ordinate axis system



Solution

3. Calculate the total force:

In the x-direction:

$$F_{T_x} = \rho Q\left(u_{2_x} - u_{1_x}\right)$$

$$u_{1_x} = u_1$$

$$u_{2_x} = u_2 \cos\theta$$

$$F_{T_x} = \rho Q\left(u_2 \cos\theta - u_1\right)$$

In the y-direction:

$$F_{T_y} = \rho Q\left(u_{2_y} - u_{1_y}\right)$$

$$u_{1_y} = u_1 \sin 0 = 0$$

$$u_{2_y} = u_2 \sin\theta$$

$$F_{T_y} = \rho Q u_2 \sin\theta$$

Solution

4. Calculate the pressure force:

$$F_p = \text{pressure force at 1 - pressure force at 2}$$

$$F_{p_x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{p_y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

5. Calculate the body force

Solution

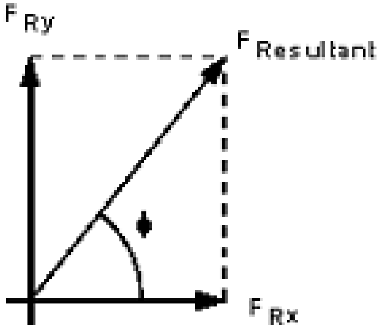
6. Calculate the resultant force:

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$

$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$F_{R_x} = F_{T_x} - F_{P_x} - 0 = \rho Q\left(u_2 \cos\theta - u_1\right) - p_1 A_1 + p_2 A_2 \cos\theta$$

$$F_{R_y} = F_{T_y} - F_{P_y} - 0 = \rho Q u_2 \sin\theta + p_2 A_2 \sin\theta$$



And the resultant force on the fluid is given by:

$$F_R = \sqrt{F_{R_x}^2 - F_{R_y}^2}$$

The direction of application is:

$$\phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right)$$

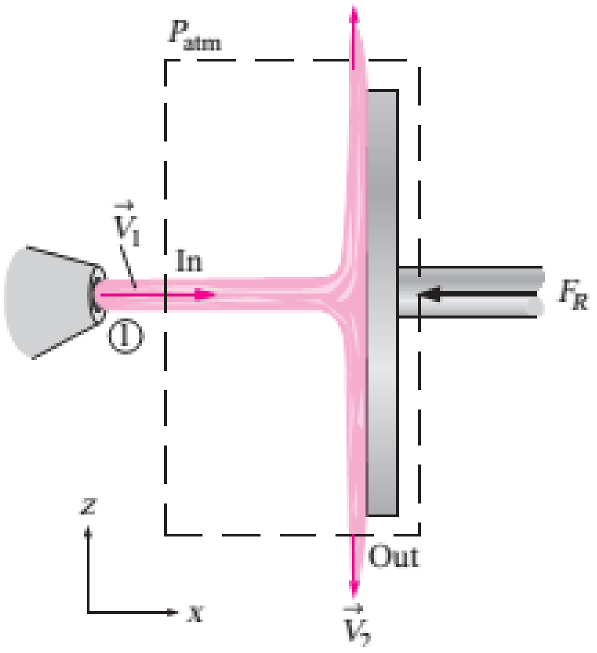
The force on the bend is the same magnitude but in the opposite direction:

$$R = -F_R$$

Example 2

Impact of a Jet on a Plane (Fixed Plane):

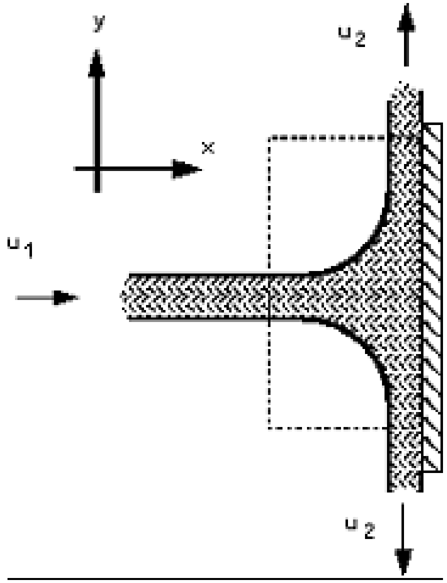
We will first consider a jet hitting a flat plate (a plane) at an angle of 90°, as shown in the figure below. We want to find the reaction force of the plate i.e. the force the plate will have to apply to stay in the same position



Solution

Step in Analysis:

1 & 2. Control volume and Co-ordinate axis are shown in the figure below.



3. Calculate the total force:

$$F_{T_x} = \rho Q\left(u_{2_x} - u_{1_x}\right)$$
$$= -\rho Q u_{1_x}$$

As the system is symmetrical the forces in the y direction cancel i.e.

$$F_{T_y} = 0$$

Solution

4. Calculate the pressure force:

The pressure force is zero as the pressure at both the inlet and the outlets to the control volume are atmospheric.

5. Calculate the body force:

As the control volume is small we can ignore the body force due to the weight of gravity.

6. Calculate the resultant force:

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$

Exerted on the fluid.

$$F_{R_x} = F_{T_x} - 0 - 0$$

$$= -\rho Q u_{1_x}$$

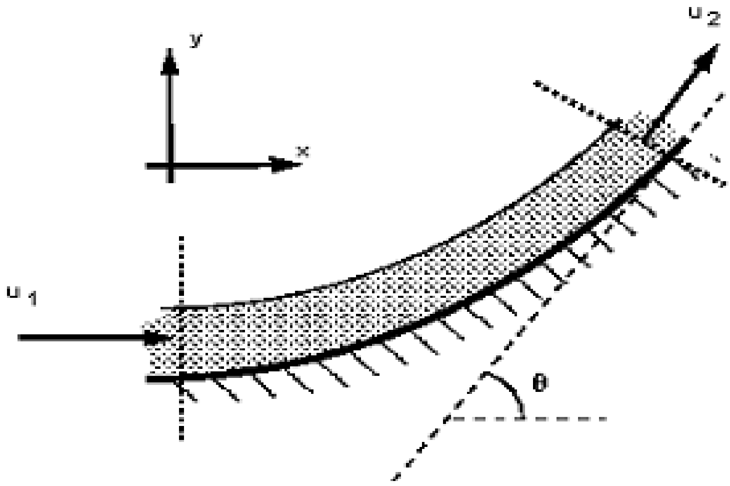
The force on the plane is the same magnitude but in the opposite direction

$$R = -F_{R_x}$$

Example 3

Force on a curved vane:

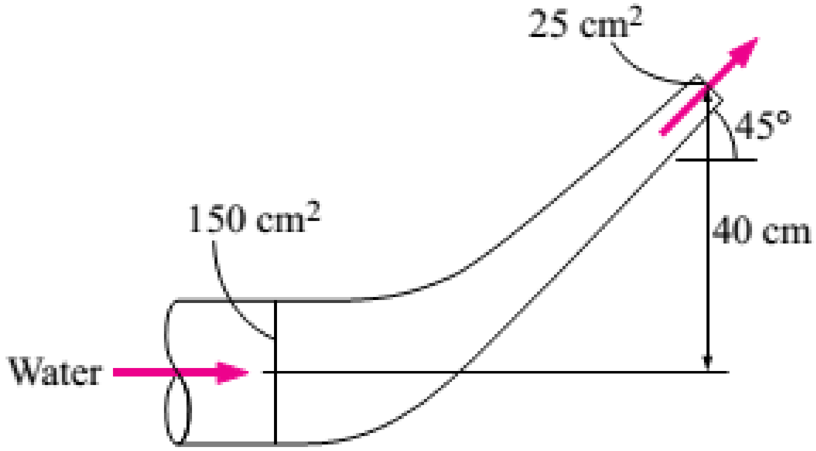
This case is similar to that of a pipe, but the analysis is simpler because the pressures are equal - atmospheric , and both the cross-section and velocities (in the direction of flow) remain constant. The jet, vane and co-ordinate direction are arranged as in the figure below.



Jet deflected by a curved vane.

Example 4

A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle θ = 45° from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.



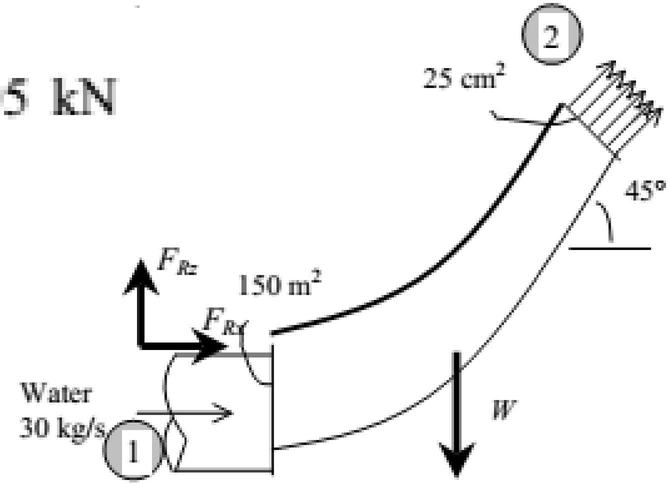
Solution

The weight of the elbow and the water in it is:

$$W = mg = (50\ \text{kg})(9.81\ \text{m/s}^2) = 490.5\ \text{N} = 0.4905\ \text{kN}$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\ \text{m}^2)} = 2.0\ \text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\ \text{m}^2)} = 12\ \text{m/s}$$



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\text{gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

$$P_{1,\text{gage}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{(12\ \text{m/s})^2 - (2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.4\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 73.9\ \text{kN/m}^2 = 73.9\ \text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$

Solution

$$F_{Rx} + P_{1,gage}A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin\theta$$

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\,gage}A_1$$

$$= 1.03(30\ \text{kg/s})[(12\cos 45° - 2)\ \text{m/s}]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$-(73.9\ \text{kN/m}^2)(0.0150\ \text{m}^2)$$

$$= -0.908\ \text{kN}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30\ \text{kg/s})(12\sin 45°\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + 0.4905\ \text{kN} = 0.753\ \text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\ kN}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = \mathbf{-39.7°}$$

Note that: **the magnitude of the anchoring force is 1.18 kN, and its line of action makes –39.7° from +x direction. Negative value for F_{Rx} indicates the assumed direction is wrong.**


Thank You
For Your Attention