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Flow-Network Design Elective Course No. 4

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## Lecture 4

## Branching Pipes

## A simple branching-pipe system

Adding a parallel pipe to the original pipe


## Example

A pipe joins two reservoirs whose head difference is 10 m . The pipe is 0.2 m diameter, 1000 m in length and has a $f$ value of 0.008 . a) What is the flow in the pipeline? b) It is required to increase the flow to the downstream reservoir by $\mathbf{3 0 \%}$. This is to be done adding a second pipe of the same diameter that connects at some point along the old pipe and runs down to the lower reservoir. Assuming the diameter and the friction factor are the same as the old pipe, how long should the new pipe be?

## Solution

## Case 1:

$$
\begin{aligned}
& \quad h_{f}=\frac{f L Q^{2}}{3 d^{5}} \\
& 10=\frac{0.008 \times 1000 Q^{2}}{3 \times 0.2^{5}} \\
& Q=0.0346 \mathrm{~m}^{3} / \mathrm{s} \\
& Q=34.6 \mathrm{litres} / \mathrm{s} \\
& \text { Case } 2: \\
& H=10=h_{f 1}+h_{f 2}=h_{f 1}+h_{f 3} \\
& \therefore \\
& h_{f 2}=h_{f 3} \\
& \quad \frac{f_{2} L_{2} Q_{2}{ }^{2}}{3 d_{2}^{5}}=\frac{f_{3} L_{3} Q_{3}{ }^{2}}{3 d_{3}^{5}}
\end{aligned}
$$

## Solution

as the pipes 2 and 3 are the same f , same length and the same diameter then $Q_{2}=Q_{3}$. By continuity $\mathrm{Q}_{1}=\mathrm{Q}_{2}+\mathrm{Q}_{3}=2 \mathrm{Q}_{2}=2 \mathrm{Q}_{3}$ So

$$
Q_{2}=\frac{Q_{1}}{2}
$$

and

$$
\mathrm{L}_{2}=1000-\mathrm{L}_{1}
$$

Then

$$
\begin{aligned}
& 10=h_{f 1}+h_{f 2} \\
& 10=\frac{f_{1} L_{1} Q_{1}^{2}}{2 d_{1}^{2}}+\frac{f_{2} L_{2} Q_{2}^{2}}{2 d_{2}^{2}} \\
& 10=\frac{f_{1} L_{1} Q_{1}^{2}}{2 d_{1}^{2}}+\frac{f_{2}\left(1000-L_{1}\right)\left(Q_{1} / 2\right)^{2}}{2 d_{2}^{2}}
\end{aligned}
$$

## Solution

As $f_{1}=f_{2}, d_{1}=d_{2}$

$$
10=\frac{f_{1} Q_{1}^{2}}{2 d_{1}^{2}}\left(L_{1}+\frac{\left(1000-L_{1}\right)}{4}\right)
$$

The new $\mathrm{Q}_{1}$ is to be $30 \%$ greater than before so $\mathrm{Q}_{1}=1.3 \times 0.034=0.442 \mathrm{~m}^{3} / \mathrm{s}$
Solve for L to give

$$
\begin{gathered}
\mathrm{L}_{1}=455.6 \mathrm{~m} \\
\mathrm{~L}_{2}=1000-455.6=544.4 \mathrm{~m}
\end{gathered}
$$

## Flow Between Reservoirs

## Problem Description:

## Given:

Reservoir elevations, sizes of pipes ,friction factor, minor loss coefficients and fluid properties
Required: Flow through each pipe and head losses Expect the flow direction in the flowing figure?


## Step analysis:

a) Apply the loss equation for each pipe: $\left(h_{L}=K Q^{2}\right)$
$\left|H_{J}-H_{A}\right|=K_{J A} Q_{J A}^{2}$
$\left|H_{J}-H_{B}\right|=K_{J B} Q_{J B}^{2}$
$\left|H_{J}-H_{C}\right|=K_{J C} Q_{J C}^{2}$

b) Continuity equation at junction $J$
net flowout of junction $J=0$
$Q_{J A}+Q_{J B}+Q_{J C}=0$
$>$ Note the sign convention: $Q_{J A}$ is the flow from $J$ to $A$; it will be negative if the flow actually goes from A to J .
$>$ The direction of flow in any pipe is always from high head to low head.
$>$ The problem and its solution method can be generalized to any number of reservoirs.

## Solution Procedure:

1. Establish the head loss vs discharge equations for each pipe.
2. Guess an initial head at the junction, $\boldsymbol{H}_{J}$.
3. Calculate flow rates in all pipes (from the head differences)
4. Calculate net flow out of J.
5. If necessary, adjust $H_{J}$ to reduce any flow imbalance and repeat from (3)

## Important Note

If the direction of flow in a pipe, say $J_{B}$, is not obvious then a good initial guess is to set $H_{J}=H_{B}$ so that there is initially no flow in this pipe. The first flow-rate calculation will then establish whether $\mathbf{H} \mathbf{J}$ should be lowered or raised and hence the direction of flow in this pipe.

## Example 1

Reservoirs A, B and C have constant water levels of 150, 120 and 90 m respectively above datum and are connected by pipes to a single junction $J$ at unknown elevation. The length ( $L$ ), diameter ( $D$ ), friction factor (f ) and minor-loss coefficient (K) of each pipe are given below. Calculate:
(a) Calculate the flow in each pipe.
(b) Calculate the pressure at the junction J .

| Pipe | $\mathrm{L}(\mathrm{m})$ | $\mathrm{D}(\mathrm{m})$ | f | K |
| :---: | :---: | :---: | :---: | :---: |
| JA | 1600 | 0.3 | 0.015 | 40 |
| JB | 1600 | 0.2 | 0.015 | 25 |
| JC | 2400 | 0.25 | 0.025 | 50 |



Prepare head loss vs discharge relations for each pipe

$$
\begin{aligned}
& \mathrm{h}_{\mathrm{L}}=\left(\frac{f L}{D}+K\right) \frac{V^{2}}{2 g} \text { where } V=\frac{4 Q}{\pi D^{2}} \\
& \mathrm{~h}_{\mathrm{L}}=\left(\frac{f L}{D}+K\right) \frac{8 Q^{2}}{\pi^{2} D^{4} g}
\end{aligned}
$$

$$
\begin{array}{ll}
\text { Pipe } A J:\left|H_{J}-H_{A}\right|=1224 Q_{N}^{2} & Q_{A}= \pm \sqrt{\frac{\left|H_{J}-150\right|}{1224}} \\
\text { Pipe } J B:\left|H_{J}-H_{B}\right|=7488 Q_{A}^{2} & Q_{J B}= \pm \sqrt{\frac{\left|H_{J}-120\right|}{7488}} \\
\text { Pipe } J C:\left|H_{J}-H_{C}\right|=6134 Q_{N}^{2} & Q_{J C}= \pm \sqrt{\frac{\left|H_{J}-90\right|}{6134}}
\end{array}
$$

The value of $H_{J}$ is varied until the net flow out of $J$ is 0 --If there is net flow into the junction then $H_{J}$ needs to be raised.
-If there is net flow out of the junction then $H_{J}$ needs to be lowered.
After the first two guesses at $\mathbf{H}_{\mathbf{J}}$, subsequent iterations are guided by interpolation.

| $\mathrm{H}_{\mathrm{J}}(\mathrm{m})$ | $\mathrm{Q}_{\mathrm{JA}}\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JB}}\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JC}}\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JA}}+\mathrm{Q}_{\mathrm{JB}}+\mathrm{Q}_{\mathrm{JC}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 120 | -0.1566 | 0.0000 | 0.0699 | -0.0867 |
| 140 | -0.0904 | 0.0517 | 0.0903 | 0.0516 |
| 132.5 | -0.1196 | 0.0409 | 0.0832 | 0.0045 |
| 131.8 | $\mathbf{- 0 . 1 2 1 9}$ | $\mathbf{0 . 0 3 9 7}$ | $\mathbf{0 . 0 8 2 5}$ | $\mathbf{0 . 0 0 0 3}$ |

The quantity of flow in each pipe is given in the bottom row of the table, with the direction implied by the sign.

$$
\begin{aligned}
& H_{J}=\frac{p_{J}}{\gamma}+Z_{J} \\
& p_{J}=\rho g\left(H_{J}-Z_{J}\right)=67000 \mathrm{~Pa}=67 \mathrm{kPa}
\end{aligned}
$$

## Examples 2

The table shows the data for the network of pipes shown conneccing four reservoirs to a common junction.

| Reservoir | Water Level <br> $(\mathrm{m})$ above a <br> datum | K for Pipe <br> connecting <br> to $\mathrm{J}\left(\mathrm{s}^{2} / \mathrm{m}^{5}\right)$ |
| :--- | :--- | :--- |
| A | 50 | 4.0 |
| B | 45 | 3.0 |
| C | 40 | 2.0 |
| D | 30 | 2.0 |



Calculate the flow in each pipe using iteration with flow accuracy less than 0.1 $\mathrm{m}^{3} / \mathrm{s}$

## Solution

Prepare head loss vs discharge relations for each pipe
$\mathrm{h}_{\mathrm{L}}=\left(\frac{f L}{D}+K\right) \frac{V^{2}}{2 g} \quad$ where $V=\frac{4 Q}{\pi D^{2}}$
$\mathrm{h}_{\mathrm{L}}=\left(\frac{f L}{D}+K\right) \frac{8 Q^{2}}{\pi^{2} D^{4} g}=K_{t} Q^{2}$
Pipe $A J:\left|H_{J}-H_{A}\right|=4 Q_{N}^{2}$

$$
Q_{N}= \pm \sqrt{\frac{\left|H_{J}-50\right|}{4}}
$$

Pipe JB: $\left|H_{J}-H_{B}\right|=3 Q_{A}^{2}$

$$
Q_{J B}= \pm \sqrt{\frac{\left|H_{J}-45\right|}{3}}
$$

Pipe JC: $\left|H_{J}-H_{C}\right|=2 Q_{A}^{2}$

$$
Q_{J C}= \pm \sqrt{\frac{\left|H_{J}-40\right|}{2}}
$$

Pipe $J C:\left|H_{J}-H_{D}\right|=2 Q_{A}^{2}$

$$
Q_{J C}= \pm \sqrt{\frac{\mid H_{J}-30}{2}}
$$

## Solution

## Assume $\mathrm{H}_{\mathrm{J}}=(50+45+40+30) / 4=41.25 \mathrm{~m}$

| $\mathrm{H}_{\mathrm{J}}(\mathrm{m})$ | $\mathrm{Q}_{\mathrm{JA}}$ <br> $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JB}}$ <br> $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JC}}$ <br> $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JD}}$ <br> $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Q}_{\mathrm{JA}}+\mathrm{Q}_{\mathrm{JB}}+\mathrm{Q}_{\mathrm{Jc}}+\mathrm{Q}_{\mathrm{JD}}$ <br> $\left(\mathrm{m}^{3} / \mathrm{s}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 41.25 | -1.479 | -1.118 | 0.791 | 3.354 | 1.548 |
| 40 | -1.581 | -1.291 | 0 | 2.236 | -0.636 |
| 40.4 | -1.549 | -1.238 | 0.447 | 2.28 | 0.06 |

## Thank You <br> For Your Atte iton

