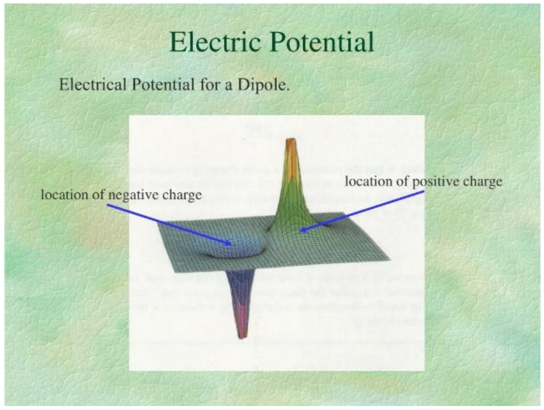
Chapter 4: Electric Potential



The electrostatic potential in the plane of an electric dipole. The potential due to each charge is proportional to the charge and inversely proportional to the distance from the charge.

Learning Objectives
After completing this chapter you should be able to

4.1 Identify that the electric force is conservative and thus has an associated potential energy.

4.2 Identify that at every point in a charged object's electric field, the object sets up an electric potential V, which is a scalar quantity that can be positive or negative depending on the sign of the object's charge.

4.3 For a charged particle placed at a point in an object's electric field, apply the relationship between the object's electric potential V at that point, the particle's charge q, and the potential energy U of the particle–object system..

4.4 Convert energies between units of joules and electron-volts.

4.5 If a charged particle moves from an initial point to a final point in an electric field, apply the relationships between the change ΔV in the potential, the particle's charge q, the change ΔU in the potential energy, and the work W done by the electric force.

4.6 If a charged particle moves between two given points in the electric field of a charged object, identify that the amount of work done by the electric force is path independent.

4.7 If a charged particle moves through a change V in electric potential without an applied force acting on it, relate V and the change K in the particle's kinetic energy.

4.8 If a charged particle moves through a change V in electric potential while an applied force acts on it, relate V, the change K in the particle's kinetic energy, and the work Wapp done by the applied force.

Key Ideas:

- The electric potential V at a point P in the electric field of a charge particle is

$$\Delta V = -\frac{W_\infty}{q_0} = \frac{U}{q_0}$$

1. The physics beyond the subject

The electrostatic force is ***conservative***, and thus we can introduce a ***potential energy (U)*** and thus we can use the powerful techniques of energy conservation in solving problems.

2. Electric Potential Energy

Consider a charge q_0 in an electric field E. The electric field exerts a force $q_0\vec{E}$ on the charge. And If you want to move the charge, you must exert a force $-q_0\vec{E}$ on it. Now, if you move the charge by a displacement $d\vec{s}$, the work done by the electric field is

$$dW = q_0\vec{E}\cdot d\vec{s} \qquad (4.1)\,,$$

and ***you*** do work equal to

$$dW_{ext} = -q_0\vec{E}\cdot d\vec{s} \qquad (4.2);$$

That is $$W_{ext} = -W \qquad (4.3)$$

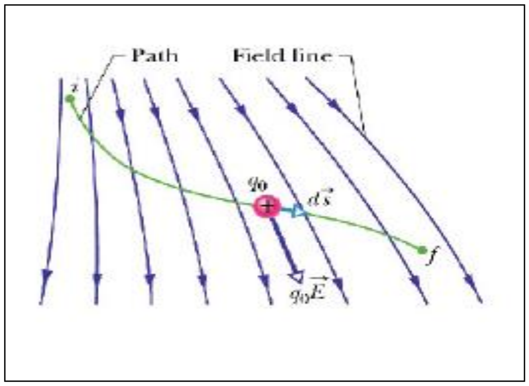


Figure 1

The total work ***<u>you</u>*** done in moving the charge from point (i) to point (f) is equal to the change in electric potential energy (ΔU), so

$$\Delta U = U_f - U_i = W_{ext} \qquad (4.4)$$

Then using (4.3) we have

$$\Delta U = -W \qquad (4.5)$$

so

$$\Delta U = -q_0\int_i^f \vec{E}\cdot d\vec{s} \quad (4.6)$$

3 Electric Potential

It is useful to define the electric potential V;

$$V = \frac{U}{q_0} \qquad (4.7)$$

$$\Delta V = \frac{\Delta U}{q_0} = -\frac{W}{q} \qquad (4.8)$$

The change in electric potential (ΔV) (potential difference) between two points (i) and (f) is given by

And from eq. (4.6) then

$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} \qquad (4.9)$$

If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential V_i must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q_0} = -\int_\infty^f \vec{E} \cdot d\vec{s} \qquad (4.10)$$

Here W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point *f*.

The Si unit of U is : Joule (J)

The Si unit of V is : Joule (J/C)

Another unit of V is : Volt (V) : (1V= 1J/ C)

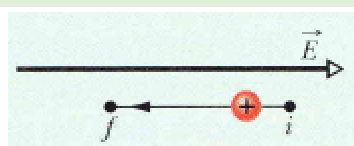
This unit of volt allows us to adopt a more conventional unit for the electric field, *E*, which is expressed in newtons per coulomb.

$$1\ \text{N/C} = \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1\ \text{V}\cdot\text{C}}{1\ \text{J}}\right)\left(\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right)$$
$$= 1\ \text{V/m}.$$

We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One *electron-volt (eV)* is the energy equal to the work required to move a single elementary charge *e*, such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

$$1\ \text{eV} = e(1\ \text{V})$$
$$= (1.60 \times 10^{-19}\ \text{C})(1\ \text{J/C}) = 1.60 \times 10^{-19}\ \text{J}.$$

CHECKPOINT
In the figure, a proton moves from point i to point f in a uniform electric field directed as shown,
(a) Does the electric field do positive or negative work on the proton?
(b) Does the electric potential energy of the proton increase or decrease



Answer: (a) negative (b) Increase

4 Equipotential Surfaces

If a charged particle moves perpendicular to electric field lines, then no work is done on it ($W = 0$), and so, its electric potential energy does not change ($\Delta U = 0$), and the electric potential does not change ($\Delta V = 0$). So;

The electric potential energy (U) and the electric potential (V) remain constant along this path.

This path in two dimensional is called equipotential line, and in three dimension is called equipotential surface.

Note that

- the electric field $(\vec{E})$ must be everywhere ***perpendicular*** to the equipotential surface.
- As equipotential surfaces are ***close together***, the E field is large

Equipotential surface and E lines for a uniform field, a point charge, and a dipole are shown in Figure 24.3.

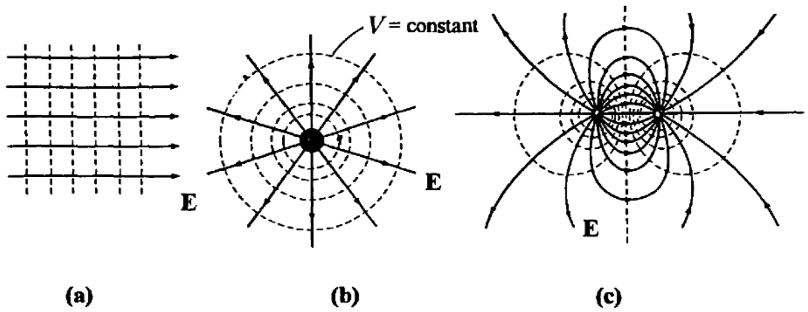


Figure 2

Example 1

Figure 3 shows electric field $\vec{E} = 150N/C$ directed downward what is the change in potential energy (ΔU) of a released electron, when the electrostatic force causes it to move vertically upward through a distance d=520 m.

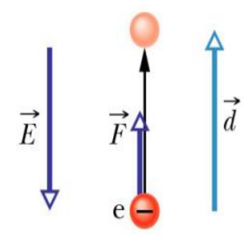


Figure 3

Solution

$$\Delta U = U_f - U_i = -q_0 \int_i^f \vec{E} \cdot d\vec{s} \qquad (4.1)$$

$$\Delta U = -(-1.6 \times 10^{-19}\ C)\int_i^f Eds\cos\theta, \qquad (\theta = 0)$$

$$= -(-1.6 \times 10^{-19}\ C)(-1)E\int_0^{150} ds, \qquad (E = 150\ N/C = constant)$$

$$= (-1.6 \times 10^{-19}\ C)(150\ N/C)(150 - 0) = -1.2 \times 10^{-4}\text{J}$$

Example 2

(a) Figure 4a shows two points i and f in a uniform electric field E, and are separated by a distance d. Find the potential difference V_f - V_i by moving a positive test charge q_0 from i to f along the path (a) which is parallel to the field direction, and along the path (b)

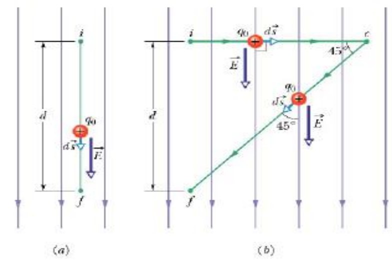


Figure 4

Solution

along path (a), from eq. 4.9, then

$$V_f - V_i \ = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$= -\int_{\boldsymbol{i}}^{\boldsymbol{f}} Eds\cos\theta \ \ (\theta = 0 \ and \ E = constant)$$

$$= -(-1)E\int_i^f ds$$

$$= (-1)E\int_0^d ds$$

$$= -Ed$$

along path (b), from eq. 2.2, then

$$V_f - V_i \ = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$= -\int_{\boldsymbol{i}}^{\boldsymbol{c}} Eds\cos 90^o \ - \int_{\boldsymbol{c}}^{\boldsymbol{f}} Eds\cos 45^o$$

$$= 0 - E\cos 45^o \int_c^f ds$$

$$= -E\cos 45^o \frac{d}{sin45^o}$$

$$= -Ed$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them (*Electrostatic force is conservative*).

5- Electric potential due to a point charge

In figure 5 Consider a point P at distance R from a fixed particle of positive charge q. To use Eq. (4.3), we imagine that we move a positive test charge q_0 from point P to infinity.

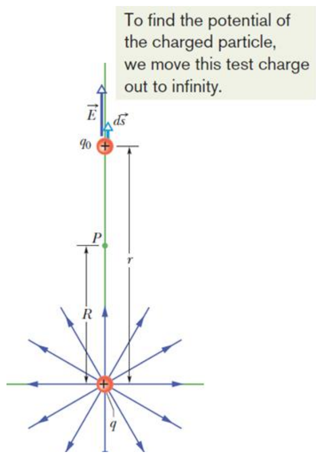


Figure 5

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V_f - V_i \ = -\int_R^f \vec{E} \cdot d\vec{s}$$

Substituting by E and with the integration we have;

$$V_f - V_i = -\int_R^f \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$
$$V_f - V_i = -\int_R^f \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$
$$V_f - V_i = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^\infty$$

$r_f = \infty$, and therefore $V_f = 0$. One therefore obtains an absolute value for the potential at any point;

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}\right)$$

switching R to r ,we then have

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right) \quad (4.11) \qquad (point\ charge)$$

Note that :

- This equation is hold also for negative charge. in which case q, is a negative sign.
- Positive charge causes positive potential , and negative charge causes negative potential.

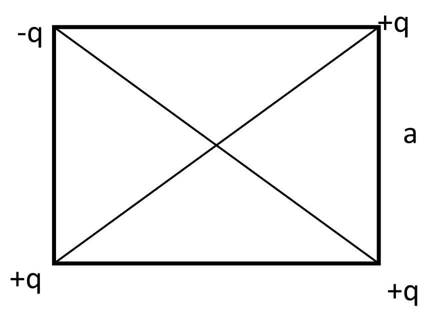
6: Potential Due to a Group of Point Charges

The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

$$\boxed{V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad n \text{ point charges}} \qquad (4.12)$$

Example 3:

A charge -q is placed at one corner of a square of side a, and charges +g are placed at each of the other corners. What is the potential at the center of the square?

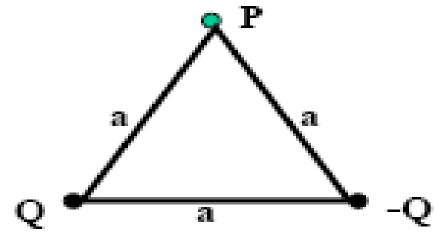


Solution:

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{a/\sqrt{2}} + \frac{q}{a/\sqrt{2}} + \frac{q}{a/\sqrt{2}} - \frac{q}{a/\sqrt{2}}\right) = \frac{\sqrt{2}}{2\pi\epsilon_0}\frac{q}{a}$$

Example 4

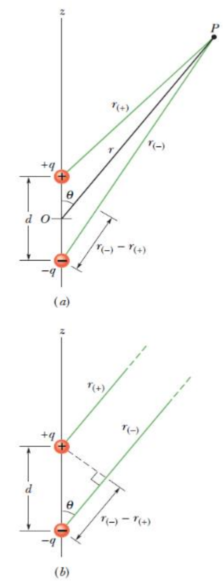
As shown in the figure, find the potential at P



Solution

$$V(P) = V_1 + V_2 = \frac{kQ}{a} + \left(-\frac{kQ}{a}\right) = 0$$

7 Potential Due to an Electric Dipole



The potential due to an electric dipole far from the dipole is given as.

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

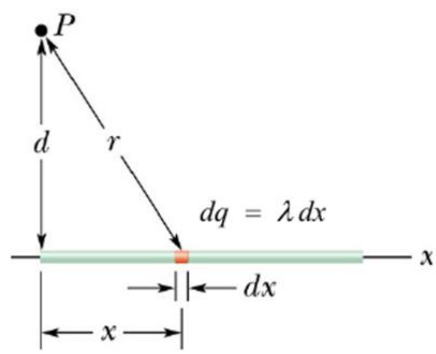
$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \qquad (4.13)}$$

8 Potential due to a continuous charge distribution.

1- Line of charge:

A rod of length L has a uniform linear charge density λ. Determine the potential at the point P, a perpendicular distance d from the left end of the rod.



Solution:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{\left(x^2 + d^2\right)^{1/2}}$$

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{\left(x^2 + d^2\right)^{1/2}} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{\left(x^2 + d^2\right)^{1/2}}$$

Using the integral tables, we find that;

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + (x^2 + d^2)^{1/2}\right)\right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L + (L^2 + d^2)^{1/2}\right) - \ln d\right].$$

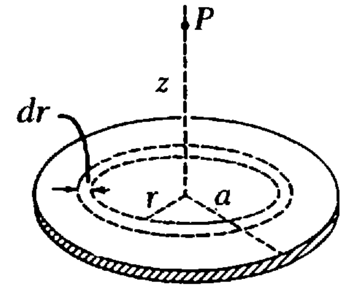
We can simplify this result by using the general relation ln A- ln B= ln(A/B). We then find

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

Because V is the sum of positive values of dV, it too is positive, consistent with the logarithm being positive for an argument greater than 1.

2- charged disk:

A disk of radius a carries a uniform surface charge density a. Find the potential on the axis at point P a distance z from the center.



$$dV = \frac{k\ dq}{\sqrt{r^2 + z^2}} = \frac{k\sigma 2\pi r\ dr}{\sqrt{r^2 + z^2}}$$

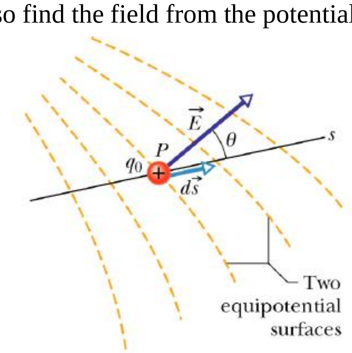
$$V = \int_0^a \frac{k\sigma 2\pi r\ dr}{\sqrt{r^2 + z^2}} = k\sigma 2\pi \int_0^a \frac{r\ dr}{\sqrt{r^2 + z^2}} = k\sigma 2\pi[\sqrt{r^2 + z^2}\ ]|_0^a$$

$$V = k\sigma 2\pi[\sqrt{a^2 + z^2} - z] \qquad k = \frac{1}{4\pi\epsilon_0} \qquad \boxed{4.14}$$

9. Calculation the field from the potential.

the Equation 4.3 shows how to find the potential if the field is given. One can also find the field from the potential, as follow.

Figure 4- shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV. As the figure suggests, the field at any point P is perpendicular to the equipotential surface through P.



Suppose that a positive test charge q_0 moves through a displacement from one equipotential surface to the adjacent surface. From Eq. 4-8, we see that the work the electric field does on the test charge during the move is $-q_0 dV$, and from eq. 4.1 , we saw also that the work done may be written as; $dW = q_0 \vec{E} \cdot d\vec{s}$. Equating the two expressions we find that;

$$-q_0 dV = q_0 \vec{E} \cdot d\vec{s}$$

Or

$$\vec{E} = -\frac{\partial V}{\partial \vec{s}} \qquad \boxed{4.15}$$

If we take the *s* axis to be, in turn, the *x*, *y*, and *z* axes, we find that the *x*, *y*, and *z* components of at any point are

$$\boxed{E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}}$$

$\partial/\partial x$ is a partial derivative that means we take the derivative of V with respect to x while holding y and z constant. In vector notation, we get

$$\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathrm{i}} + \frac{\partial V}{\partial y}\hat{\mathrm{j}} + \frac{\partial V}{\partial z}\hat{\mathrm{k}}\right) \qquad (4.16)$$

For the simple situation in which the electric field is uniform, We can approximate;

$$\mathrm{E} = -\frac{\Delta E}{\Delta s} \ (4.17)$$

<u>Example 5</u> : Suppose, in some region of space, the electric potential is given as; $V(x) = Ax^2$, where A constant. What are the x, y, and z components of the electric field in this region.

<u>Solution:</u>

$$E_x = -\frac{d}{dx}V = -A\frac{dx^2}{dx} = -2Ax$$

$$E_y = -\frac{d}{dy}V = -A\frac{dx^2}{dy} = 0$$

and also $\quad E_z = -\dfrac{d}{dz}V = 0$

Example 7: As we know, the electric potential at any point on the central axis of a uniformly charged disk is given by

$$V = k\sigma 2\pi[\sqrt{a^2 + z^2} - z] \qquad k = \frac{1}{4\pi\epsilon_0}$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk

Solution:

Circular symmetry leads to

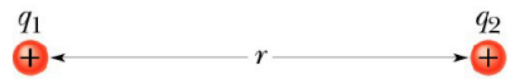
$$E_x = E_y = 0 \qquad E_z = -\frac{\partial V}{\partial z}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}\left(\sqrt{z^2 + R^2} - z\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

10 : Electric potential energy of a system of point charge.

If a charge q_2 is placed in the field of a charge q_1 a distance r away, the potential energy of the pair is then;



$$\boxed{U = q_2 V_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r}} \qquad 4.18$$

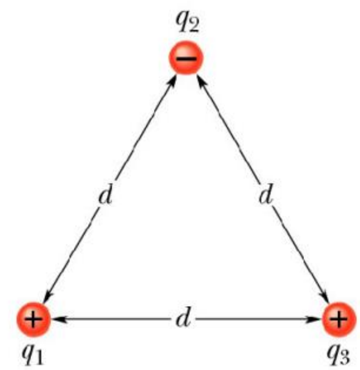
The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

If there are more than two charges present, the electrostatic energy of the system is

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)} \qquad 4.19$$

Example 6

The Figure shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that d =12 cm and that



$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

SOLUTION

$$\begin{aligned} U &= U_{12} + U_{13} + U_{23} \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right) \\ &= -\frac{10q^2}{4\pi\varepsilon_0 d} \\ &= -\frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(10)(150 \times 10^{-9}\,\mathrm{C})^2}{0.12\,\mathrm{m}} \\ &= -1.7 \times 10^{-2}\,\mathrm{J} = -17\,\mathrm{mJ}. \qquad \text{(Answer)} \end{aligned}$$

11 Potential of a Charged isolated Conductor

We have seen that the charge on a conductor resides on the outer surface and that the electric field within the conductor is zero. This means that the potential everywhere (inside and on the surface) a conductor ***is constant***.